Quantitative methods

Week #8-9

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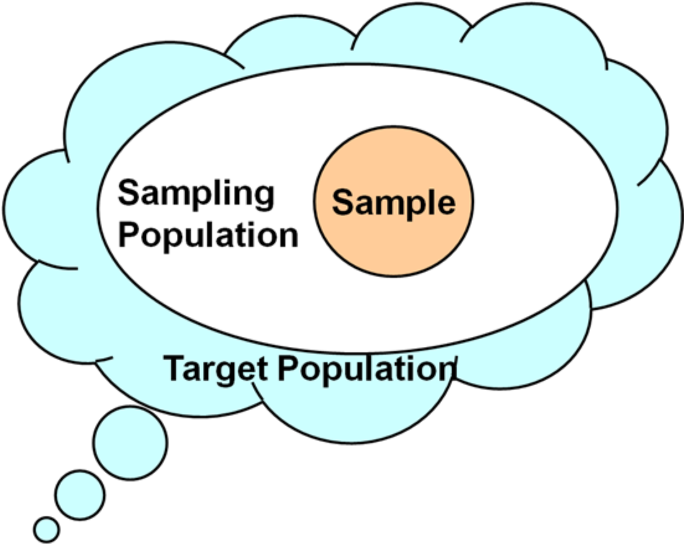
Outline

Sampling theory

Elements



Sampling methods - Probability sampling

A short summary

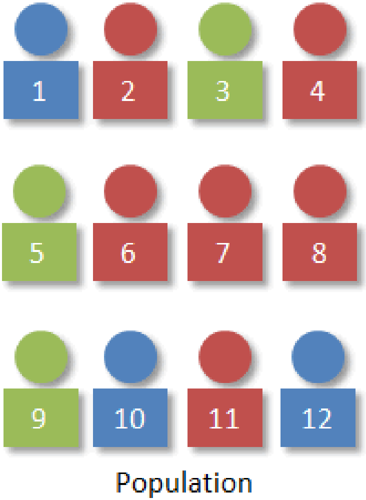
Probability sampling:

1. Simple Random Sampling,
2. Stratified Random Sampling,
3. Systematic Random Sampling,
4. Cluster (Area) Random Sampling,
5. Multi-Stage Sampling.

A subset of the population.

Simple Random Sampling

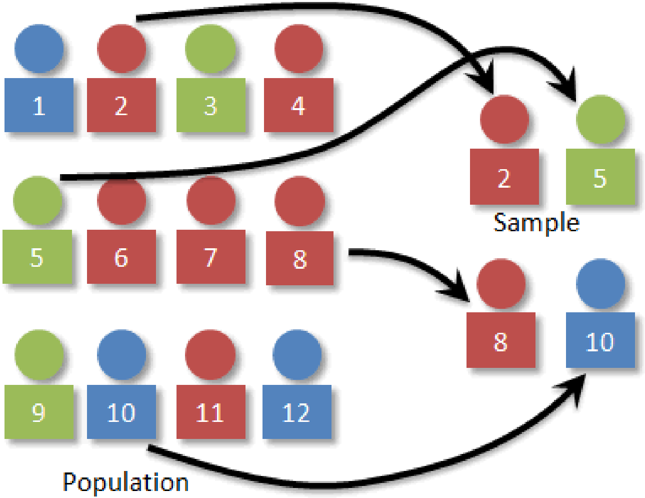
Drawing a sample



Source: Dan Kerlner, Elgin Community College

Simple Random Sampling

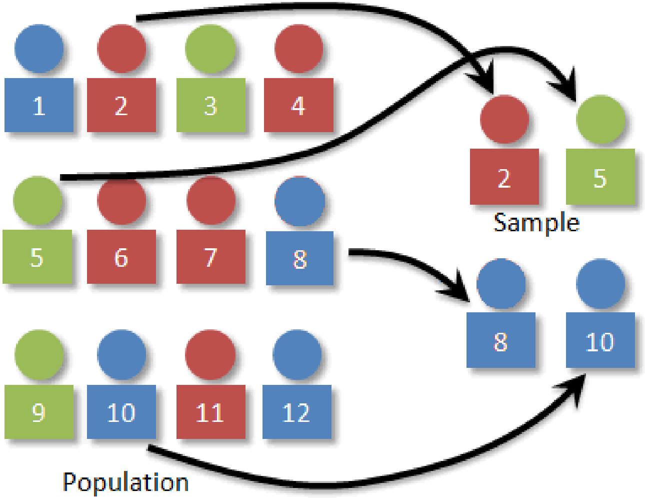
Drawing a sample



Source: Dan Kerlner, Elgin Community College

Simple Random Sampling

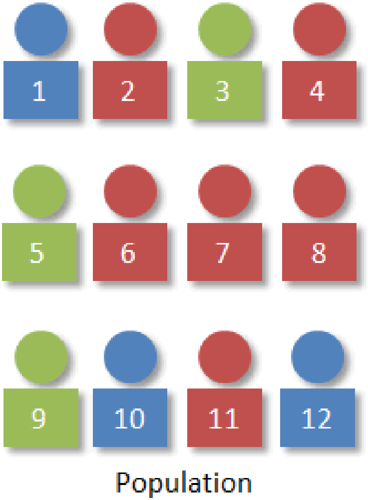
Drawing a sample



Source: Dan Kerlner, Elgin Community College

Systematic Random Sampling

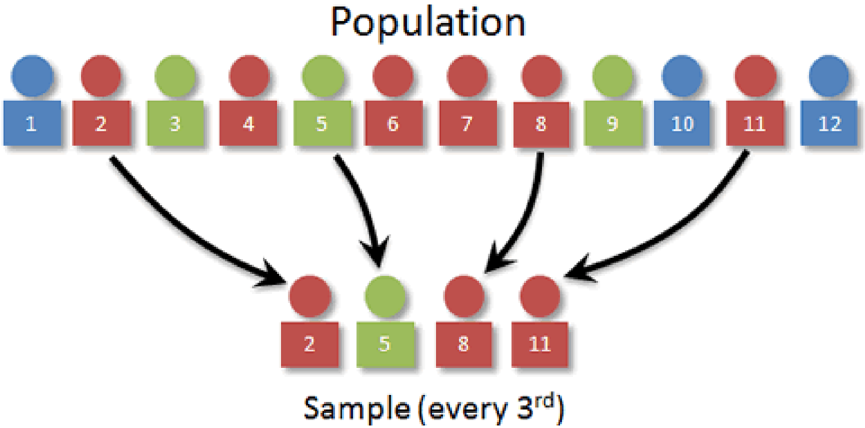
Drawing a sample



Source: Dan Kerlner, Elgin Community College

Systematic Random Sampling

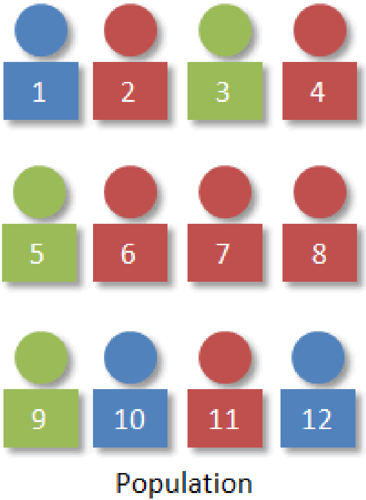
Drawing a sample



Source: Dan Kerlner, Elgin Community College

Stratified Sampling

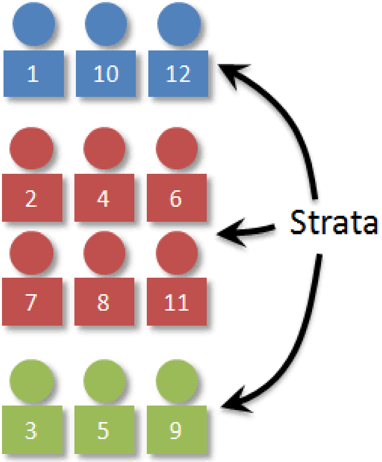
Drawing a sample



Source: Dan Kerlner, Elgin Community College

Stratified Sampling

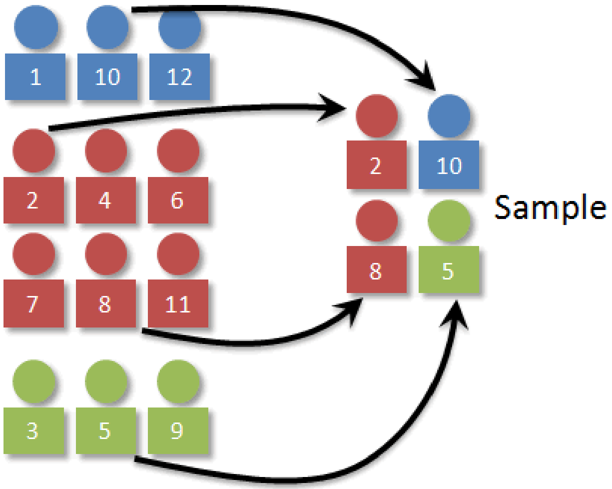
Drawing a sample



Source: Dan Kerlner, Elgin Community College

Stratified Sampling

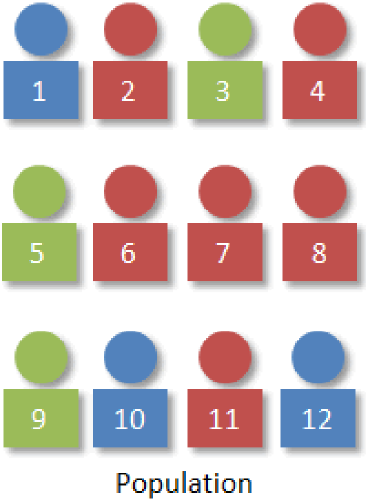
Drawing a sample



Source: Dan Kerlner, Elgin Community College

Systematic+Stratified Random Sampling

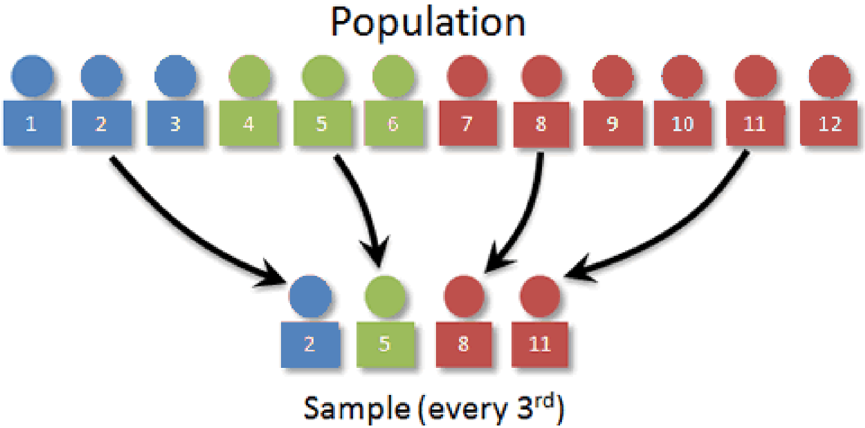
Drawing a sample



Source: Dan Kerlner, Elgin Community College

Systematic+Stratified Random Sampling

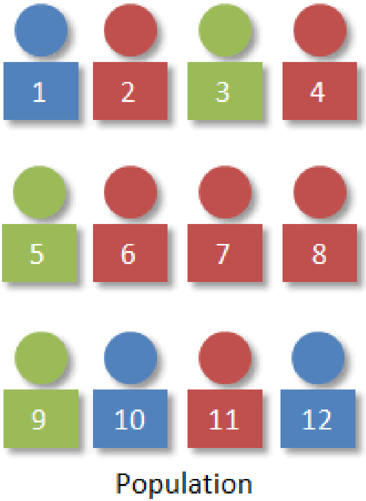
Drawing a sample



Source: Dan Kerlner, Elgin Community College

Multi-Stage Sampling

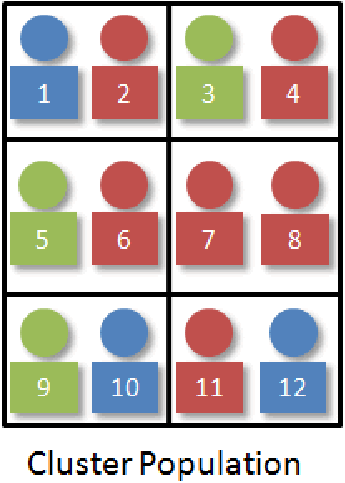
Drawing a sample



Source: Dan Kerlner, Elgin Community College

Multi-Stage Sampling

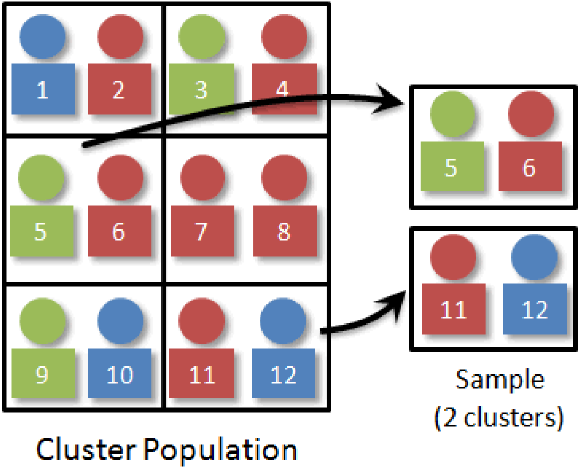
Drawing a sample



Source: Dan Kerlner, Elgin Community College

Multi-Stage Sampling

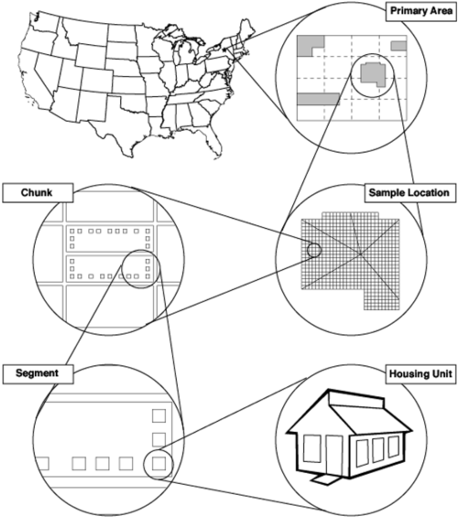
Drawing a sample



Source: Dan Kerlner, Elgin Community College

Cluster Sampling

Drawing a sample

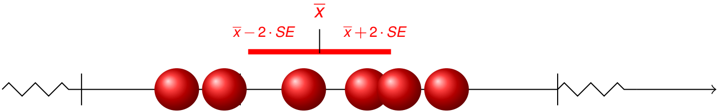


For Simple Random Sampling:

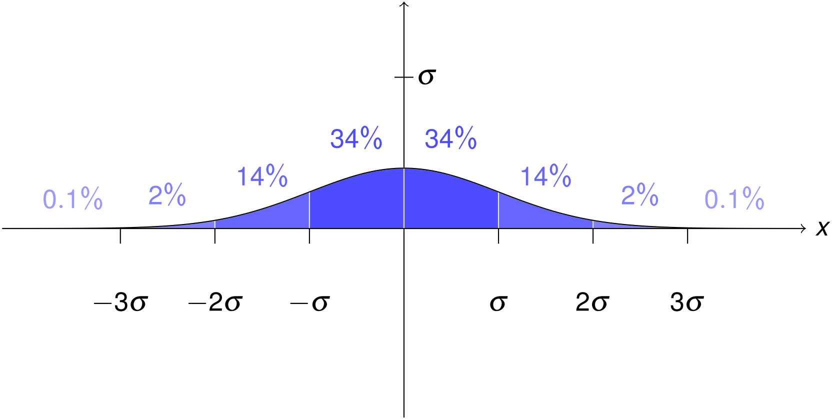
- mean: $\overline{x} = \frac{\sum_{i=1}^{n} x_i}{n}$
- standard deviation: $\sigma = \sqrt{\sum_{i=1}^{n} \frac{(x_i - \overline{x})^2}{n}}$
- standard error: $SE = \frac{\sigma}{\sqrt{n}} \cdot FPC$
- Finite Population Correction: if sampling fraction is large (>5%)

$$FPC = \sqrt{1 - \frac{n}{N}}$$

- confidence interval: $\overline{x} \pm z \cdot SE$, where $z = 1,96$
- confidence interval: $[\overline{x} - 2 \cdot SE; \overline{x} + 2 \cdot SE]$



$$\frac{1}{\sigma\sqrt{2\pi}}\exp\left(\frac{-x^2}{2\sigma^2}\right)$$

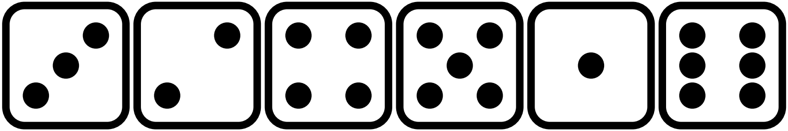


standard normal distribution: $\bar{x} = 0, \sigma = 1$

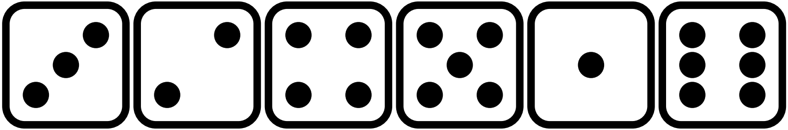
Game rules

Roll the dice!
If the result is even, the player wins the rolled value in dollars.
If the result is odd, the playes pays 2 dollars to the bank.

After rolling the below values, what would you think about the expected value of the game?



Would you continue playing?



$$X = \{-2, 2, 4, -2, -2, 6\}$$

$$\bar{x} = \frac{-2+2+4+2+2+6}{6} = \frac{6}{6} = \frac{1}{1} = 1$$

$$\sigma = \sqrt{\frac{(-2-1)^2 + (2-1)^2 + (4-1)^1 + (-2-1)^1 + (-2-1)^2 + (6-1)^2}{5}} =$$

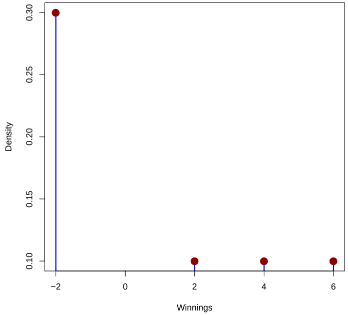
$$= \sqrt{\frac{9+1+9+9+9+25}{5}} = \sqrt{\frac{62}{5}} = \sqrt{12.4} = 3.521363$$

$$SE = \frac{3.521363}{\sqrt{6}} = \frac{3.521363}{2.44949} = 1.437591$$

The expected value can vary between -1.87 and 3.87 at 95% CI.

Good luck!

Forget about the experiment and try to determine the **real** expected value of the game!



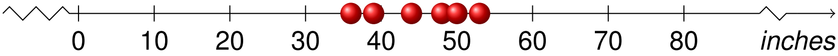
What is wrong with the above plot?

The height, in inches, of six trees at a nursery are shown at the specificed dates.

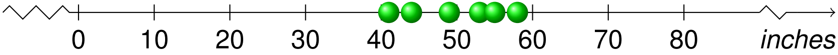
Find the mean, standard deviation and standard error of the heights!

Is there a significant difference between the means of samples?

1. **2011 March 22**: 36 48 50 44 53 39



2. **2011 April 1**: 41 53 55 49 58 44



The height, in inches, of six trees at a nursery are shown at the specificed dates.
Find the mean, standard deviation and standard error of the heights!
Is there a significant difference between the means of samples?

1. **2011 March 22**: 36 48 50 44 53 39

$$X = \{36, 48, 50, 44, 53, 39\}$$

$$\overline{x} = \frac{36 + 48 + 50 + 44 + 53 + 39}{6} = \frac{270}{6} = 45$$

$$\sigma = \sqrt{\frac{(36-45)^2 + (48-45)^2 + (50-45)^2 + (44-45)^2 + (54-45)^2 + (39-45)^2}{5}} =$$

$$= \sqrt{\frac{81 + 9 + 25 + 1 + 64 + 36}{5}} = \sqrt{\frac{216}{5}} = \sqrt{43.2} = 6.57$$

$$SE = \frac{6.57}{\sqrt{6}} = \frac{6.57}{2.44} = 2.68$$

The expected value can vary between 40.5 and 49.5 at 95% CI.

Computation

Comparison of samples

The height, in inches, of six trees at a nursery are shown at the specificed dates.
Find the mean, standard deviation and standard error of the heights!
Is there a significant difference between the means of samples?

1. **2011 April 1**: 41 53 55 49 58 44

$$X = \{41, 53, 55, 49, 58, 44\}$$

$$\overline{x} = \frac{41+53+55+49+58+44}{6} = \frac{300}{6} = 50$$

$$\sigma = \sqrt{\frac{(41-50)^2+(53-50)^2+(55-50)^2+(49-50)^2+(58-50)^2+(44-50)^2}{5}} =$$

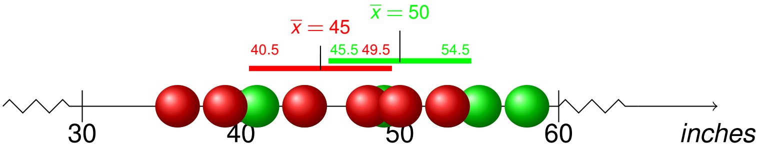
$$= \sqrt{\frac{81+9+25+1+64+36}{5}} = \sqrt{\frac{216}{5}} = \sqrt{43.2} = 6.57$$

$$SE = \frac{6.57}{\sqrt{6}} = \frac{6.57}{2.44} = 2.68$$

The expected value can vary between 45.5 and 54.5 at 95% CI.

The height, in inches, of six trees at a nursery are shown at the specificed dates.
Find the mean, standard deviation and standard error of the heights!
Is there a significant difference between the means of samples?

1. **2010 November 22**: 36 48 50 44 53 39
2. **2011 April 1**: 41 53 55 49 58 44



We have seen in the dice example, that the standard error (1.437591) could be relatively high compared to the mean (1).

If we would check the exact same values (-2, 2, 4, -2, -2, 6) denoting the temperature measured from Monday to Saturday, then would you think that the average temperature at the audited week cannot be estimated more precisely than the earlier computed confidence interval (-1.87 – 3.87)? You have only one missing data!

$$SE = \frac{\sigma}{\sqrt{n}} \cdot \sqrt{1 - \frac{n}{N}}$$

Is there any difference between computing the standard error in Hungary or in the United States?

Computation

Standard error in finite population

$$X = \{-2, 2, 4, -2, -2, 6\}$$

$$\bar{x} = \frac{-2+2+4+2+2+6}{6} = \frac{6}{6} = \frac{1}{1} = 1$$

$$\sigma = \sqrt{\frac{(-2-1)^2+(2-1)^2+(4-1)^1+(-2-1)^1+(-2-1)^2+(6-1)^2}{5}} =$$

$$= \sqrt{\frac{9+1+9+9+9+25}{5}} = \sqrt{\frac{62}{5}} = \sqrt{12.4} = 3.521363$$

$$SE = \frac{3.521363}{\sqrt{6}} \cdot FPC = \frac{3.521363}{2.44949} \cdot FPC = 1.437591 \cdot FPC$$

$$FPC = \sqrt{1-\frac{n}{N}} = \sqrt{1-\frac{6}{7}} = 0.377$$

$$SE = 0.54$$

The expected value can vary between 0.46 and 1.54 at 95% CI (opposed to: 1.87, 3.87).

Computation

Exercise

„The gas prices dramatically increased in 2011 in Hungary. We asked drivers about how much they would pay for one litre of gasoline. The results showed that there are some drivers who would even pay more then 450 forints for a litre, others do not tend to refill at the prices of 400.”

Forensis Autóklub (November of 2011)

„How much would you pay for one litre of gas?"

410, 420, 420, 430, 500, 450, 400, 425, 460

„How much would you pay for one litre of gas?”

410, 420, 420, 430, 500, 450, 400, 425, 460

Descriptive statistics:

- **mean**: $\overline{x} = \frac{410+420+420+430+500+450+400+425+460}{9} = 435$
- **median**: 425
- **mode**: 420
- **minimum**: 400
- **maximum**: 500
- **range**: 100

Computation

Exercise

„How much would you pay for one litre of gas?"

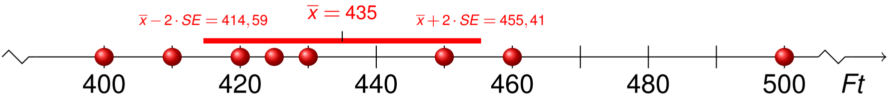
410, 420, 420, 430, 500, 450, 400, 425, 460

- **mean**: $\overline{x} = \frac{410+420+420+430+500+450+400+425+460}{9} = 435$
- **standard deviation**: $S^* = 30.619$
- **standard error**: $SE = \frac{30{,}619}{\sqrt{9}} = \frac{30{,}619}{3} = 10,206$
- **confidence interval**: $435 \pm 2 \cdot 10,206 = [414,59; 455,41]$

„How much would you pay for one litre of gas?"

410, 420, 420, 430, 500, 450, 400, 425, 460

- **mean**: $\overline{x} = \frac{410+420+420+430+500+450+400+425+460}{9} = 435$
- **standard deviation**: $S^* = 30.619$
- **standard error**: $SE = \frac{30{,}619}{\sqrt{9}} = \frac{30{,}619}{3} = 10,206$
- **confidence interval**: $435 \pm 2 \cdot 10,206 = [414,59; 455,41]$



Standard error and sampling

Examples

A módszertan haszna. EP választások 2009: „Hajszálpontos mérés”

	Nézőpont		Tárki	Medián	*NRC*	
	BSZ	BSZP	BSZP	??	??	**eredmény**
Fidesz	54%	**66%**	**70%**	60%	50%	**56,4%**
MSZP	12%	14%	17%	21%	**26%**	**17,4%**
Jobbik	6%	7%	4%	7%	**13%**	**14,8%**
MDF	5%	6%	**1%**	4%	4%	**5,3%**
SZDSZ	3%	4%	3%	4%	3%	**2,2%**

A módszertan haszna. EP választások 2009: „Hajszálpontos mérés”

	Nézőpont		Tárki	Medián	*NRC*	
	BSZ	BSZP	BSZP	??	??	**eredmény**
Fidesz	54%	**66%**	**70%**	60%	50%	**56,4%**
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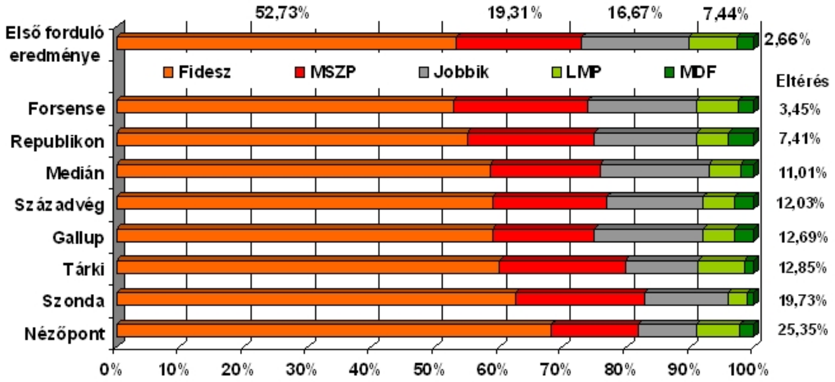
Jobbik	6%	7%	4%	7%	**13%**	**14,8%**
MDF	5%	6%	**1%**	4%	4%	**5,3%**
SZDSZ	3%	4%	3%	4%	3%	**2,2%**

	Nézőpont	TÁRKI	Medián	*NRC*
Kutatás ideje	V. 20-22.	V. 7-20	V. 22-26.	n.a.
Módszer	Telefonos lekérdezés	Személyes lekérdezés (?)	Személyes lekérdezés	Online kérdőív
Megkérdezettek száma	1000	1000	1200	1000

Source: lectures of Dr. Bartus Tamás

Standard error and sampling

Examples



Source: spss.hu

Bernoulli distribution:

- p chance for 1, q $(= 1 - p)$ chance for 0 value
- **mean**: p
- **median**: –
- **mode**: $\begin{cases} 0 & \text{if } q > p \\ 0,1 & \text{ha } q = p \\ 1 & \text{if } q < p \end{cases}$
- **standard deviation**: $\sqrt{p(1-p)}$
- **variance**: $p(1-p)$
- **standard error**: $SE = \frac{S^*}{\sqrt{n}} \cdot \sqrt{1 - \frac{n}{N}} \approx \frac{S^*}{\sqrt{n}} \approx \frac{\sqrt{p(1-p)}}{\sqrt{n}}$
- **confidence interval**: $\overline{x} \pm z \cdot SE$, where $z = 1,96$

Bernoulli distribution:

- assume the maximum of standard error,
- standard error is affected by standard deviation and sample size,
- higher sample size lowers standard error,
- higher standard deviation results in higher standard error.

Which p value would result in the maximum of standard deviation?

$$S^* = \sqrt{p(1-p)}$$

Bernoulli distribution:

- assume the maximum of standard error,
- standard error is affected by standard deviation and sample size,
- higher sample size lowers standard error,
- higher standard deviation results in higher standard error.

Which *p* value would result in the maximum of standard deviation?

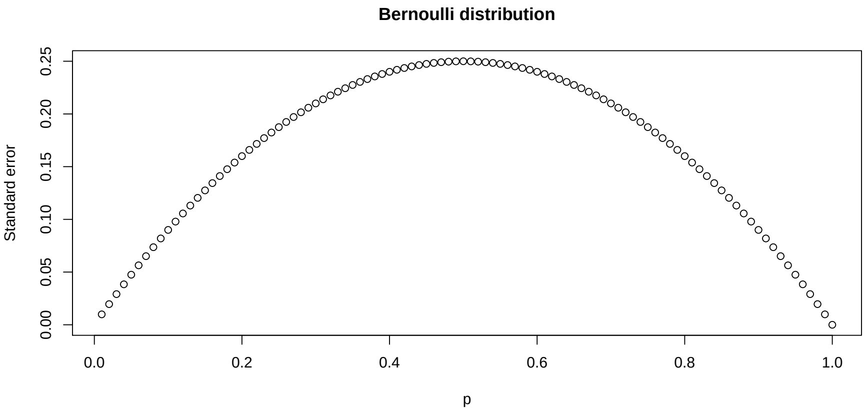
$$S^* = \sqrt{p(1-p)}$$

$$p = 0.5$$

$$VAR(x) = 0.5 \cdot (1 - 0.5) = 0.5^2 = 0.25$$

Standard error with dichotome variables

Being a pessimist



standard error: $SE = \frac{S^*}{\sqrt{n}} \cdot \sqrt{1 - \frac{n}{N}} \approx \frac{S^*}{\sqrt{n}} \approx \frac{\sqrt{p(1-p)}}{\sqrt{n}}$

Compute the sample size to measure the support for a party with the precision of 2 percent!

Standard error with dichotome variables

Determining sample size

Compute the sample size to measure the support for a party with the precision of 2 percent!

- 2 percent $=> SE = 1$,
- maximum of variance: $50 \cdot (100 - 50) = 2500$
- $SE = \frac{s^*}{\sqrt{n}}$

$$\Downarrow$$

- $1 = \frac{\sqrt{2500}}{\sqrt{n}}$

$$\Downarrow$$

- $1 \cdot \sqrt{n} = \sqrt{2500}$
- $n = 2500$

Compute the sample size to measure the time spent in front of television among Hungarian citizents! Let us choose a precision of 5 minutes.

Compute the sample size to measure the time spent in front of television among Hungarian citizents! Let us choose a precision of 5 minutes.

- 5 mins $=>$ $SE = 2.5$,
- estimated standard deviation: 10
- $SE = \frac{S^*}{\sqrt{n}}$

$$\Downarrow$$

- $2,5 = \frac{10}{\sqrt{n}}$

$$\Downarrow$$

- $2,5 \cdot \sqrt{n} = 10$
- $\sqrt{n} = 4$
- $n = 16$

Compute the sample size to measure the time spent in front of television among Hungarian citizents! Let us choose a precision of 1 minutes.

Compute the sample size to measure the time spent in front of television among Hungarian citizents! Let us choose a precision of 1 minutes.

- 1 mins $=>$ $SE = 0.5$,
- estimated deviation: 10
- $SE = \frac{S^*}{\sqrt{n}}$

$$\Downarrow$$

- $0,5 = \frac{10}{\sqrt{n}}$

$$\Downarrow$$

- $0,5 \cdot \sqrt{n} = 10$
- $\sqrt{n} = 20$
- $n = 400$

We asked 4 student about the number of cats at home:

	Rockers	Rappers
Girls	9	7
Boys	3	1

Imagine, what would be the results if the sample was choosen randomly and if it was stratified?

Choosing samples of n=2:

1. SRS: 6 possible samples: (1,7) (1,9) (3,7) (3,9) (1,3) (7,9)
$\overline{x} = \frac{4+5+5+6+2+8}{6} = 5, S^* = \frac{1+0+0+1+9+9}{6} = 3.33$
2. Strat. Sampling: 4 possible samples: (1,7) (1,9) (3,7) (3,9)
$\overline{x} = \frac{4+5+5+6}{4} = 5, S^* = \frac{1+0+0+1}{4} = 0.5$
3. Strat. Sampling: 4 possible samples: (1,3) (1,9) (3,9) (3,7)
$\overline{x} = \frac{2+5+6+5}{4} = 4.5, S^* = \frac{2.5^2+0.5^2+1.5^2+0.5^2}{4} = 2.25$

Nonprobability sampling:

1. Accidental, Haphazard or Convenience Sampling,
2. Purposive Sampling:
    1. Modal Instance Sampling,
    2. Expert Sampling,
    3. Quota Sampling:
        1. Proportional Quota Sampling,
        2. Nonproportional Quota Sampling.
    4. Heterogeneity Sampling,
    5. Snowball Sampling.

It was a pleasure!

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