Quantitative methods

Week #10-11

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Outline

Sampling methods - Nonprobability sampling

A short summary

Nonprobability sampling:

1. Accidental, Haphazard or Convenience Sampling,
2. Modal Instance Sampling,
3. Expert (Judgmental) Sampling,
4. Quota Sampling:
    1. Proportional Quota Sampling,
    2. Nonproportional Quota Sampling.
5. Heterogeneity Sampling (deviant cases),
6. Snowball Sampling.

POPULATION	Female	Male	$\sum$
Economics	10	10	20
Sociology	40	10	50
Philoposphy	10	20	30
$\sum$	60	40	100

Stratified Sampling:

?

Quota Sampling:

?

Final examination questions

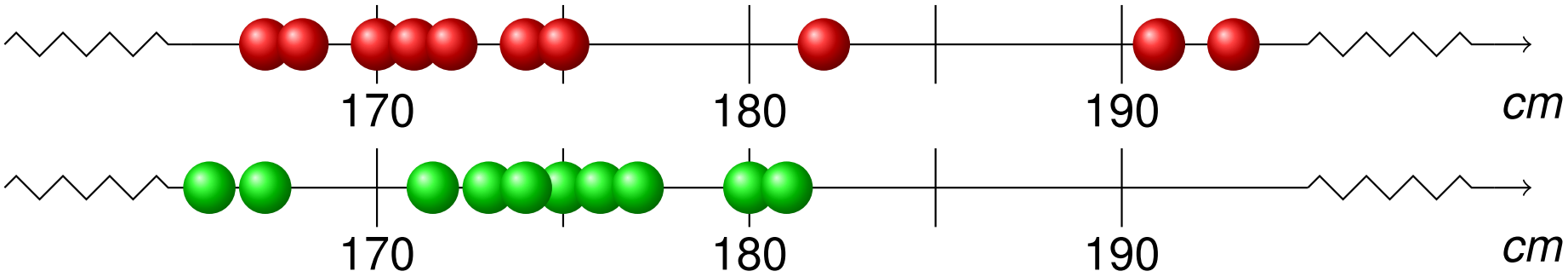
Comprehensive exam

Singleton, R. A. Jr. and Bruce C. Straits (1999): Approaches to Social Research. Third Edition. Oxford University Press: New York/Oxford.

Questions:

1. What is reliability? *How do the main rules concerning the order of survey questions improve the reliability and validity of survey data?* (pp. 113-117, 292-296)
2. **What is meant by probability sampling? How do stratification and multistage cluster sampling affect sampling errors? Why?** (pp. 141-142, 145-156)
3. **What are the main types of non-probability sampling? Explain why these types do not meet the criteria of probability samples.** (pp. 157-169)
4. **What factors affect the desired sample size?** (pp. 163-169)

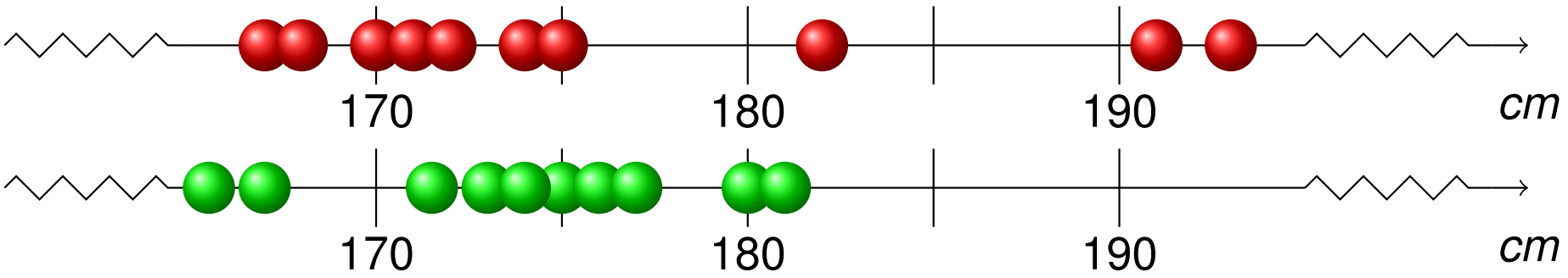
We have measured 10-10 students in two classrooms.

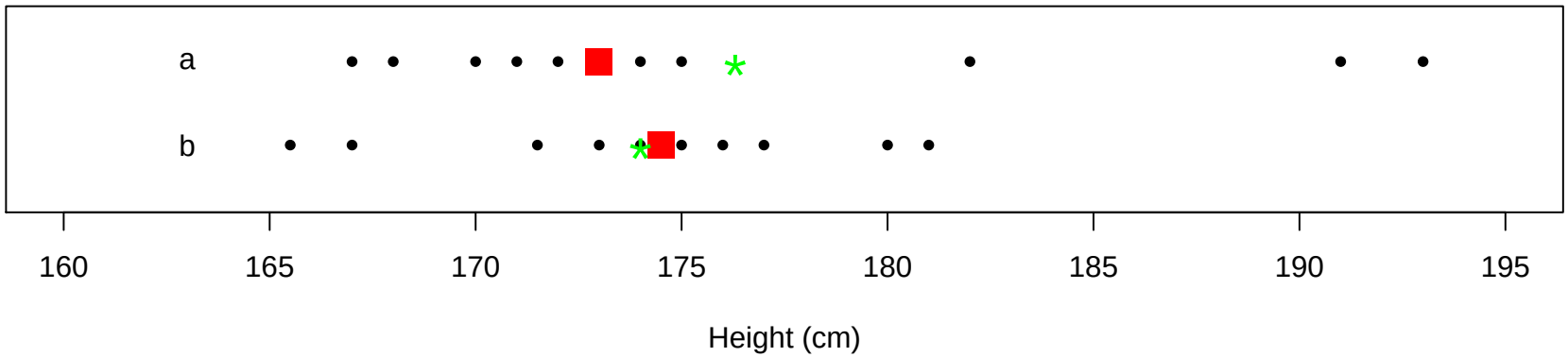


Which class has higher students based on this small sample? Think about averages as good estimates of populatioin parameters!

Repeating

Averages





Research in an elementary school

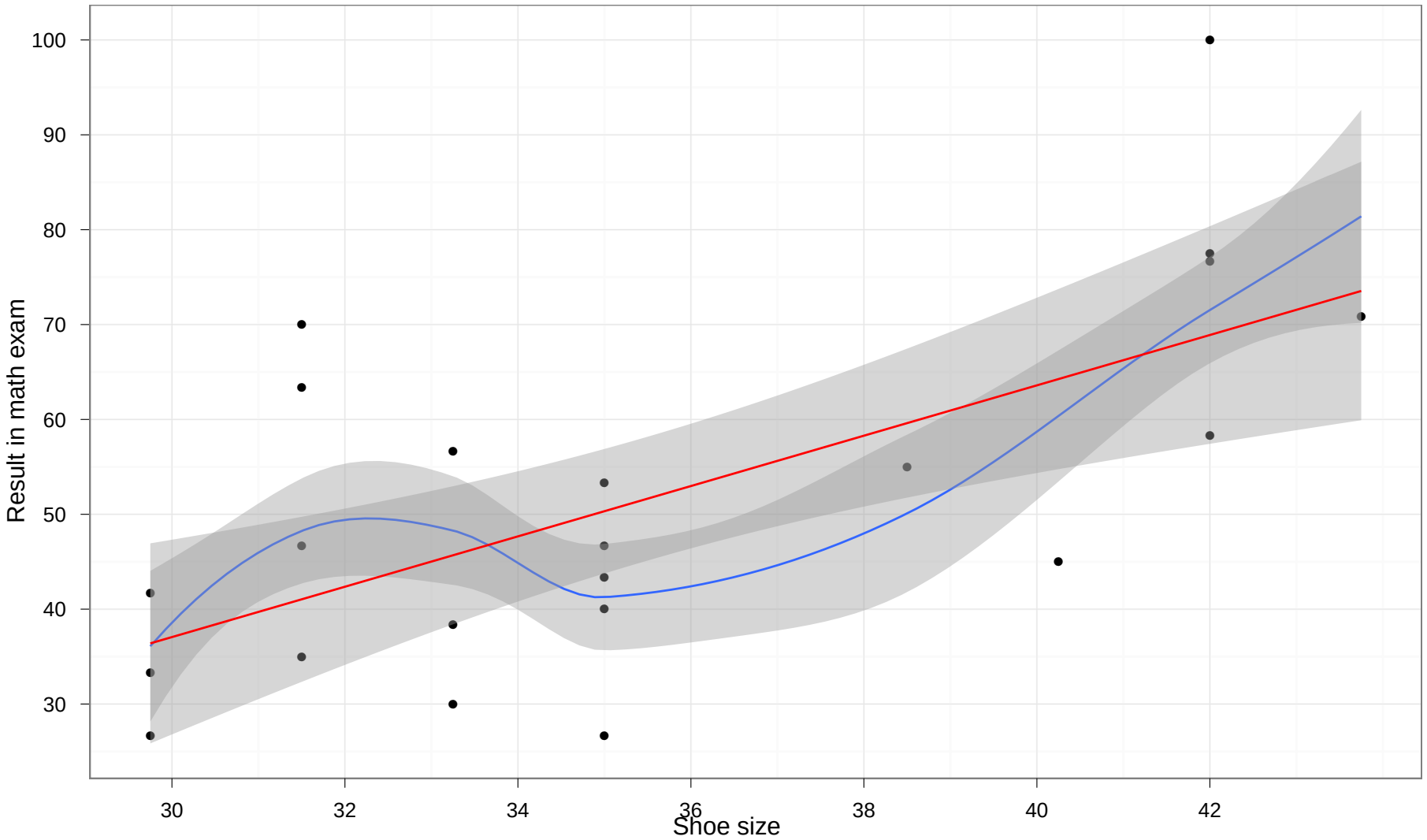
Big shoes and smart kids (example)

We made a small research on the age and shoe size of some students in an elementary shool, where we also conducted a math exam. See detailed results below:

	Shoe size	Math result
1	29.75	26.67
2	29.75	33.33
3	29.75	41.67
4	31.50	35.00
5	31.50	46.67
6	31.50	63.33
7	31.50	70.00
8	33.25	30.00
9	33.25	38.33
10	33.25	56.67
11	35.00	26.67
12	35.00	40.00
13	35.00	43.33
14	35.00	46.67
15	35.00	53.33
16	38.50	55.00
17	40.25	45.00
18	42.00	58.33
19	42.00	76.67
20	42.00	77.50
21	42.00	100.00
22	43.75	70.83

Research in an elementary school

Big shoes and smart kids (example)



Research in an elementary school

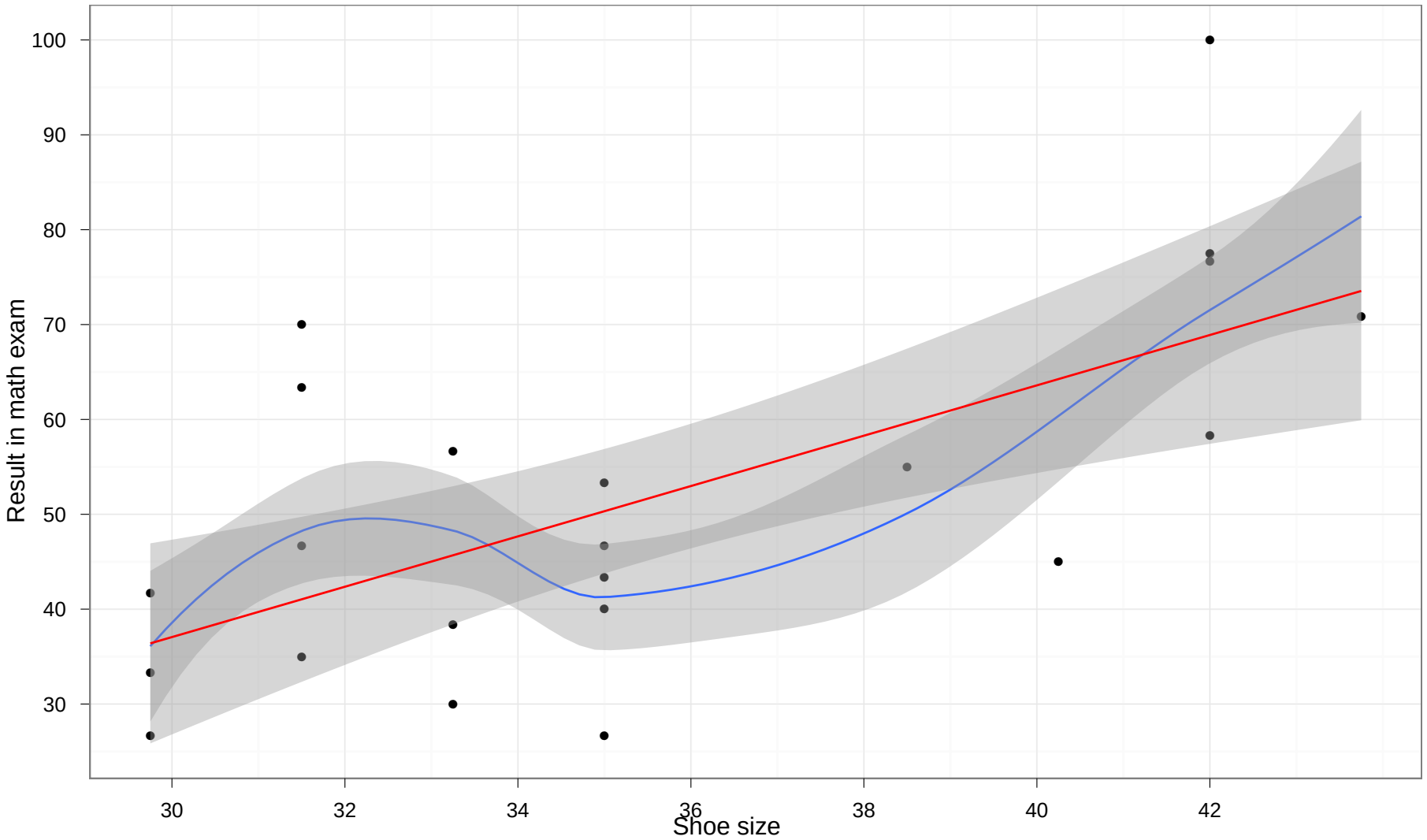
Big shoes and smart kids (example)

We made a small research on the age and shoe size of some students in an elementary shool, where we also conducted a math exam. See detailed results below:

	Shoe size	Math result	Age
1	29.75	26.67	3
2	29.75	33.33	7
3	29.75	41.67	5
4	31.50	35.00	8
5	31.50	46.67	10
6	31.50	63.33	11
7	31.50	70.00	12
8	33.25	30.00	7.
9	33.25	38.33	7
10	33.25	56.67	12
11	35.00	26.67	6
12	35.00	40.00	8
13	35.00	43.33	6
14	35.00	46.67	10
15	35.00	53.33	11
16	38.50	55.00	9
17	40.25	45.00	9
18	42.00	58.33	9
19	42.00	76.67	16
20	42.00	77.50	18
21	42.00	100.00	19
22	43.75	70.83	14

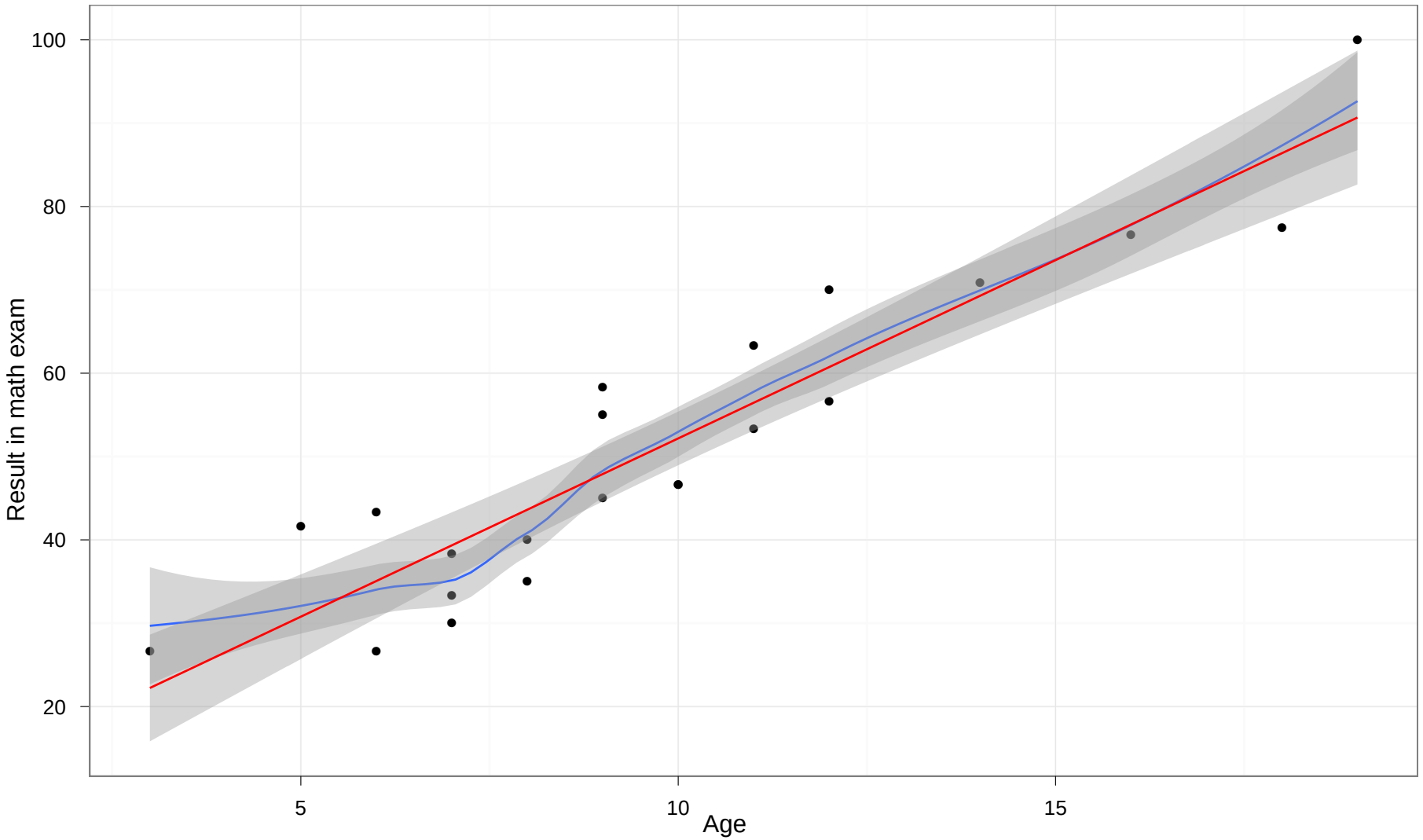
Research in an elementary school

Big shoes and smart kids (example)



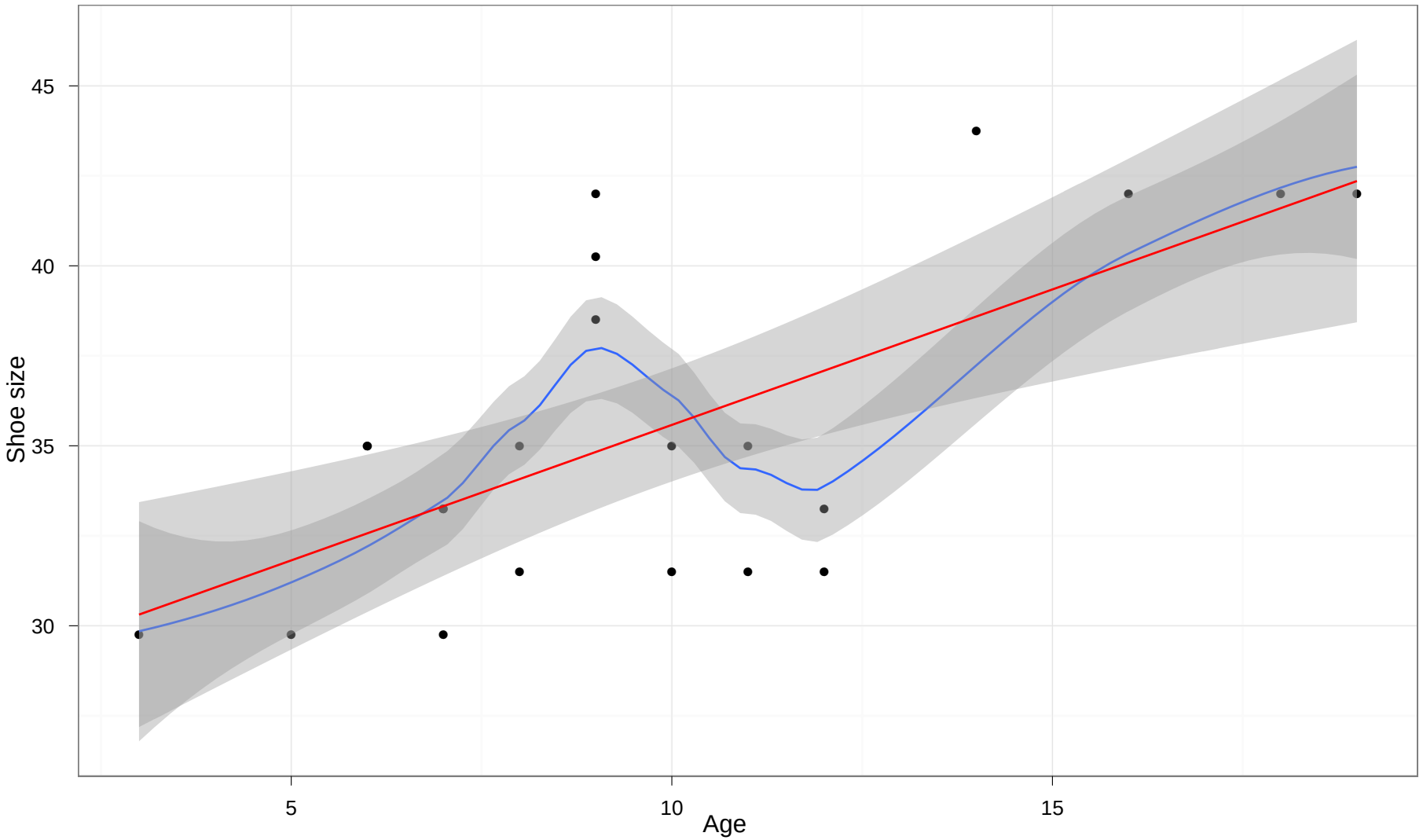
Research in an elementary school

Big shoes and smart kids (example)



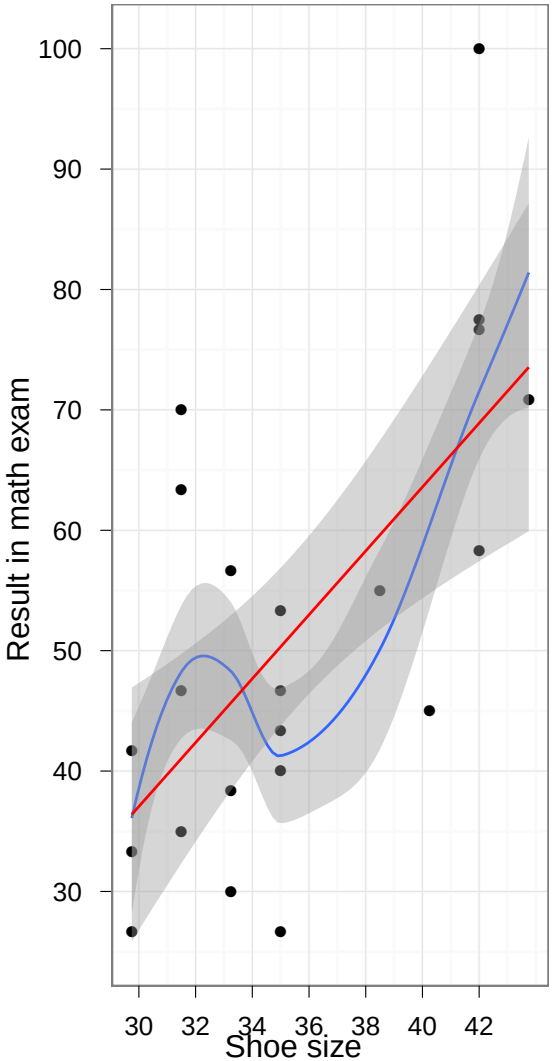
Research in an elementary school

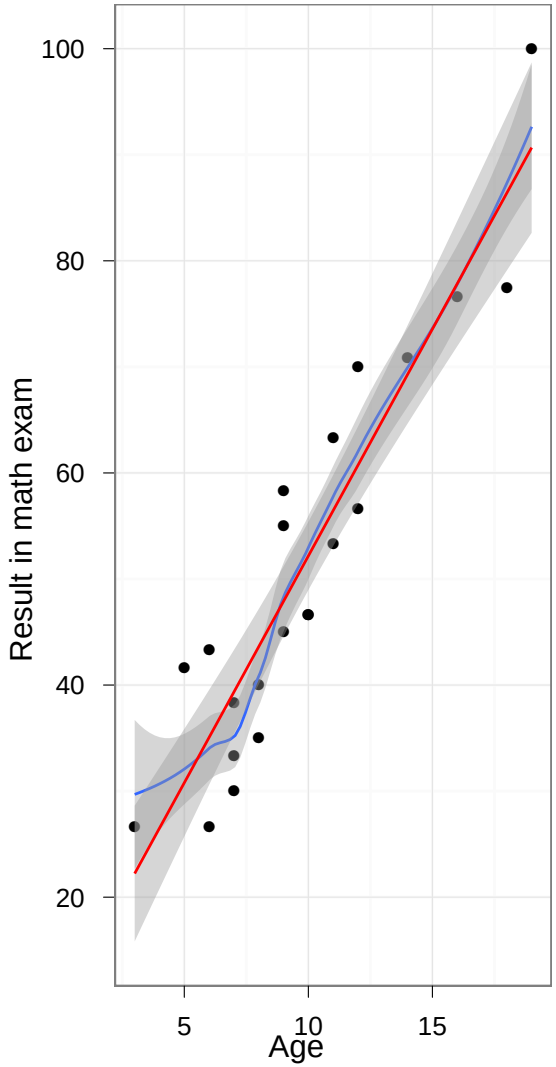
Big shoes and smart kids (example)

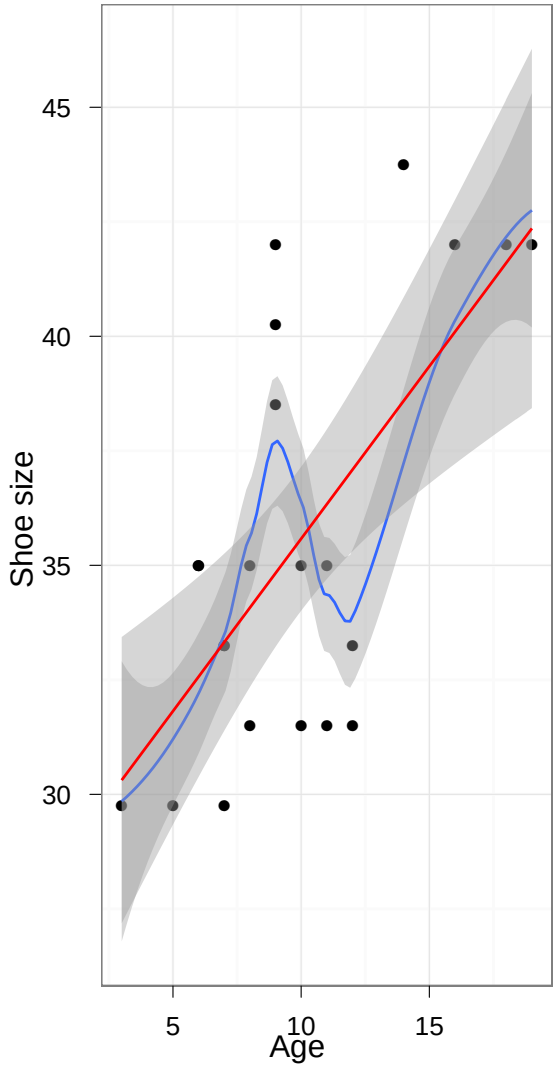


Research in an elementary school

Big shoes and smart kids (example)

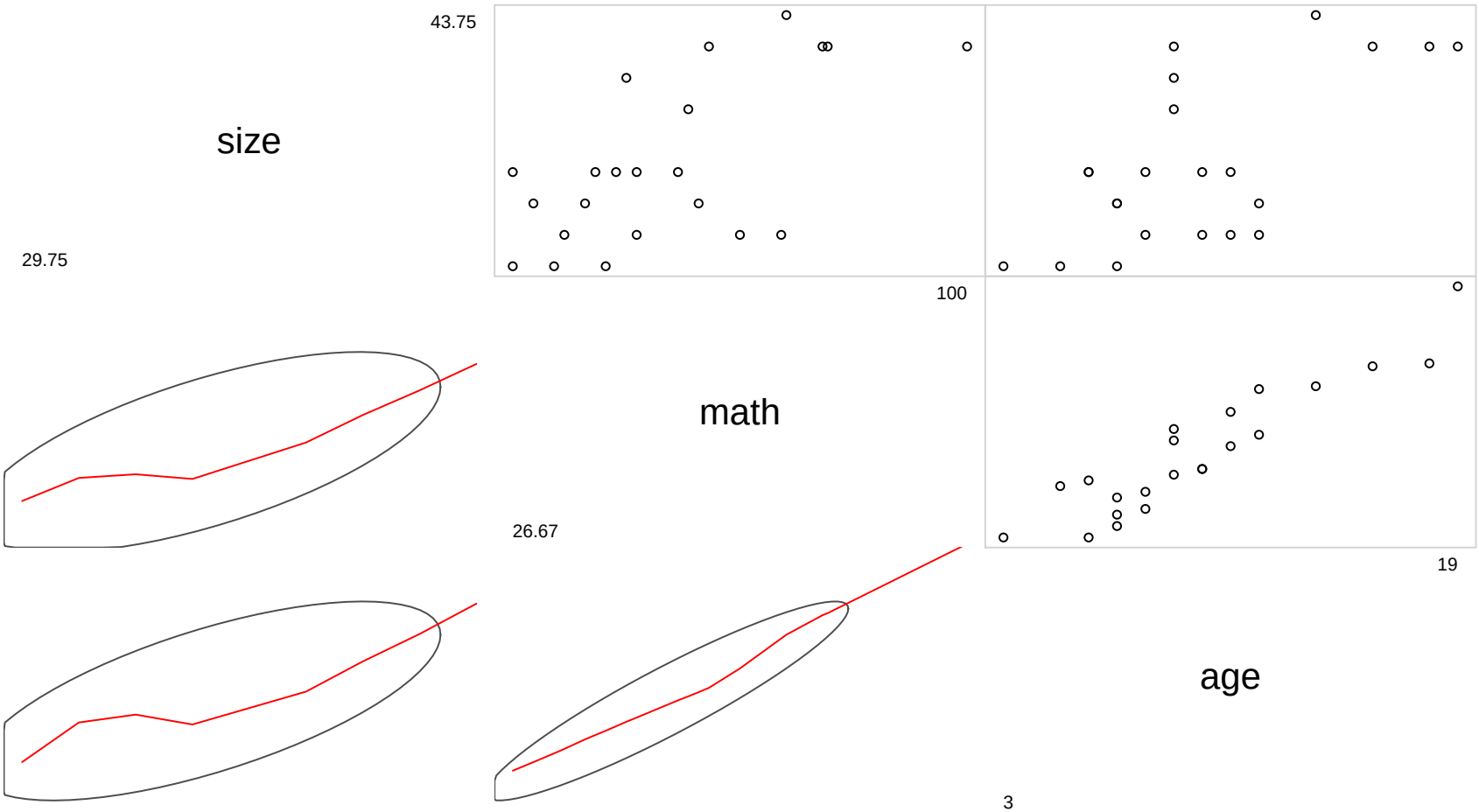






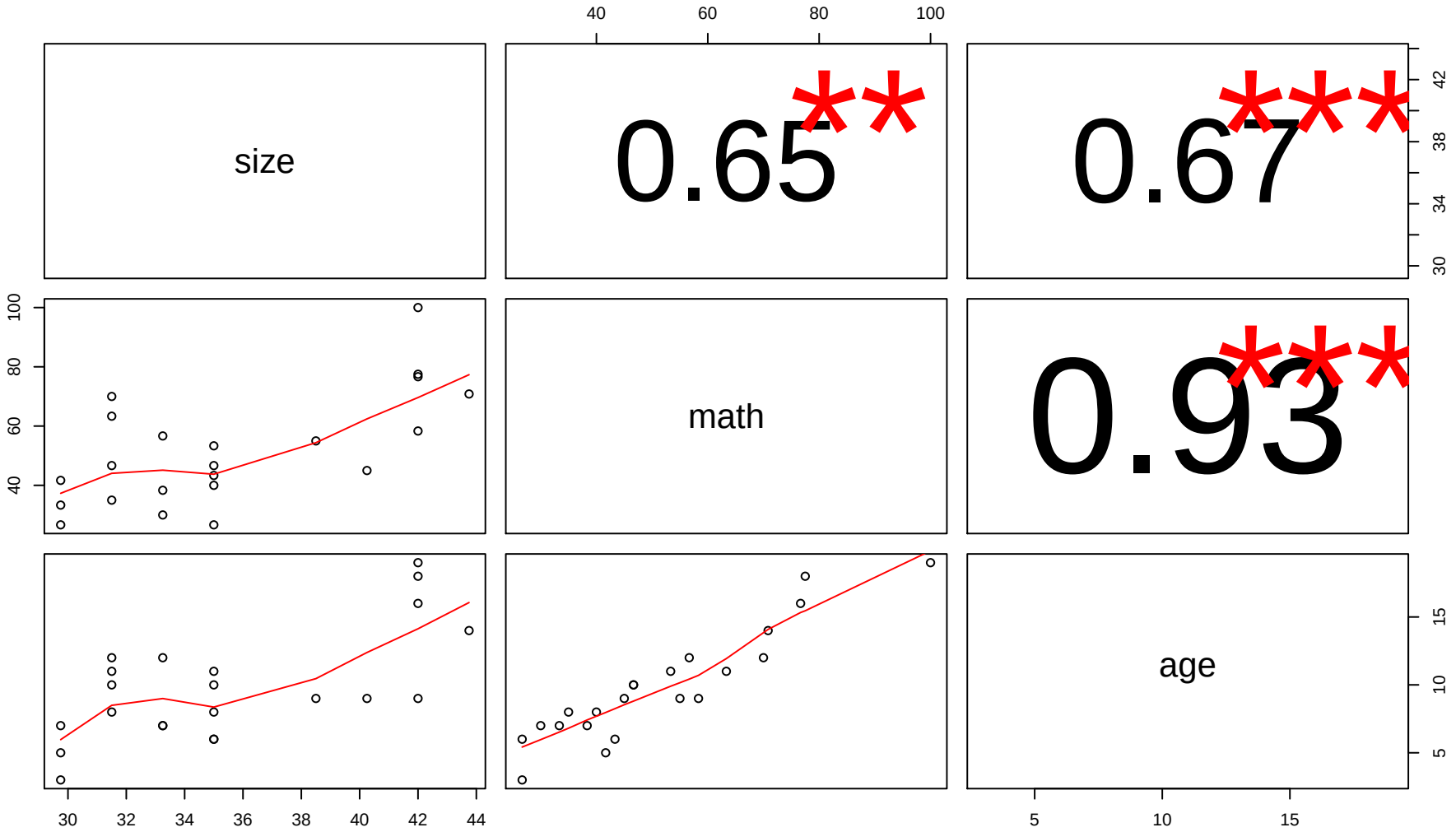
Research in an elementary school

Big shoes and smart kids (example)



Research in an elementary school

Big shoes and smart kids (example)



Partial correlation:

$$r_{math,size \cdot age} = 0.11$$

$$r_{math,age \cdot size} = 0.87$$

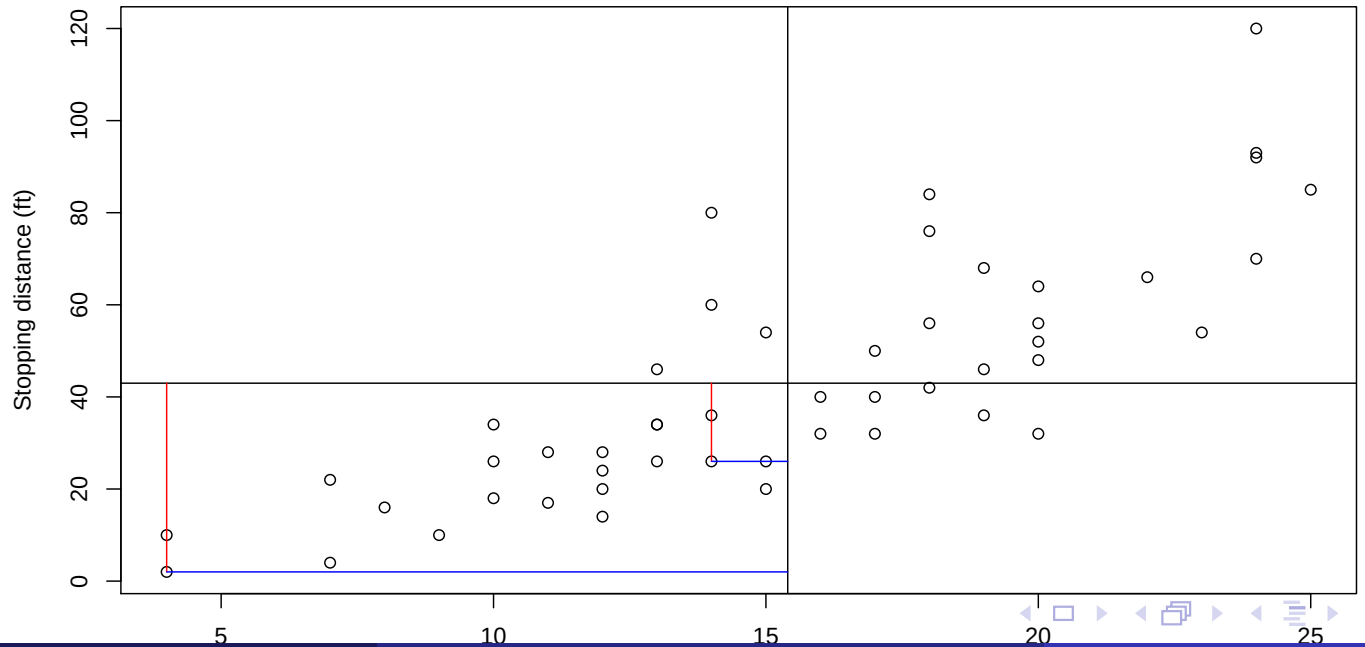
$$r_{size,age \cdot math} = 0.22$$

For x and y variables the joint variablity could be computed by :

$$COV(xy) = \sum_{i=1}^{n} \frac{(x_i - \overline{x})(y_i - \overline{y})}{n-1}$$

$$remember : \sigma = \sqrt{\sum_{i=1}^{n} \frac{(x_i - \overline{x})^2}{n}}$$

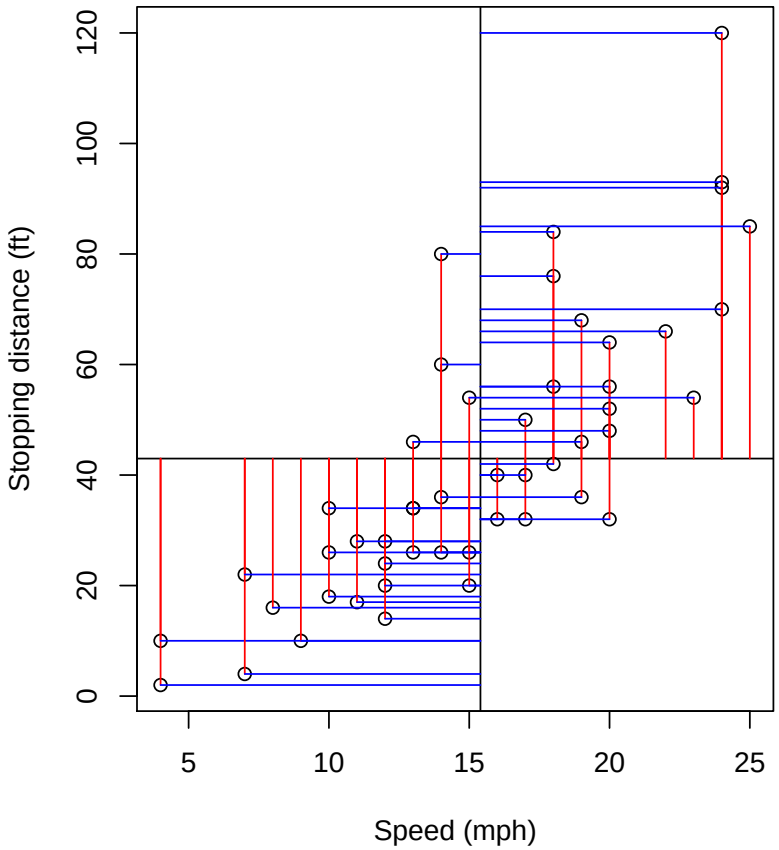
Ezekiel, M. (1930) Methods of Correlation Analysis. Wiley.



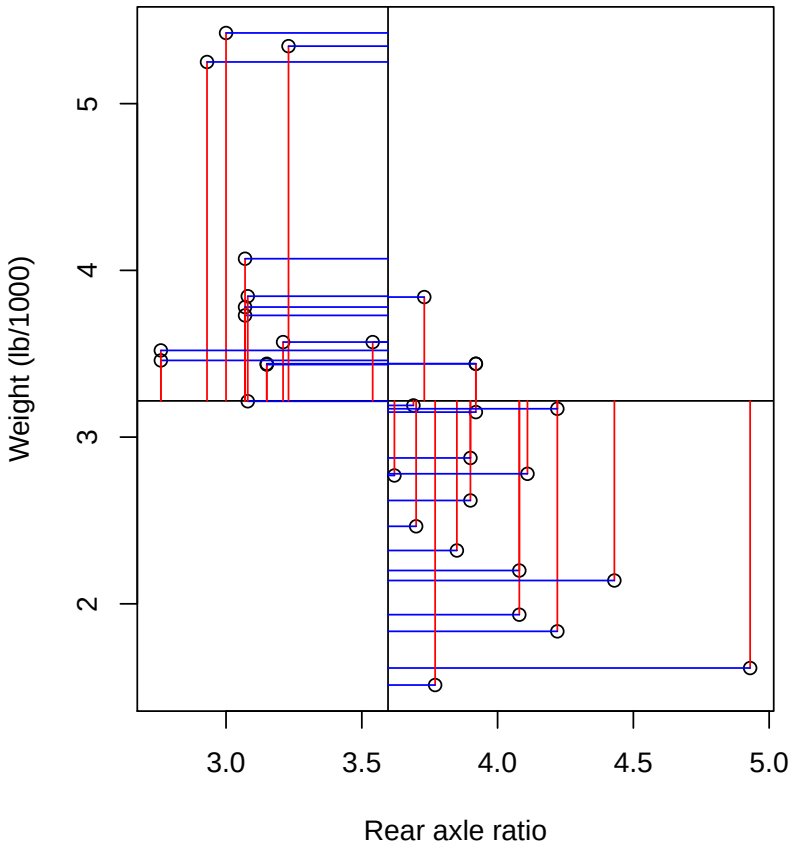
Theoretical background

Covariation

Ezekiel, M. (1930):
Methods of Correlation Analysis



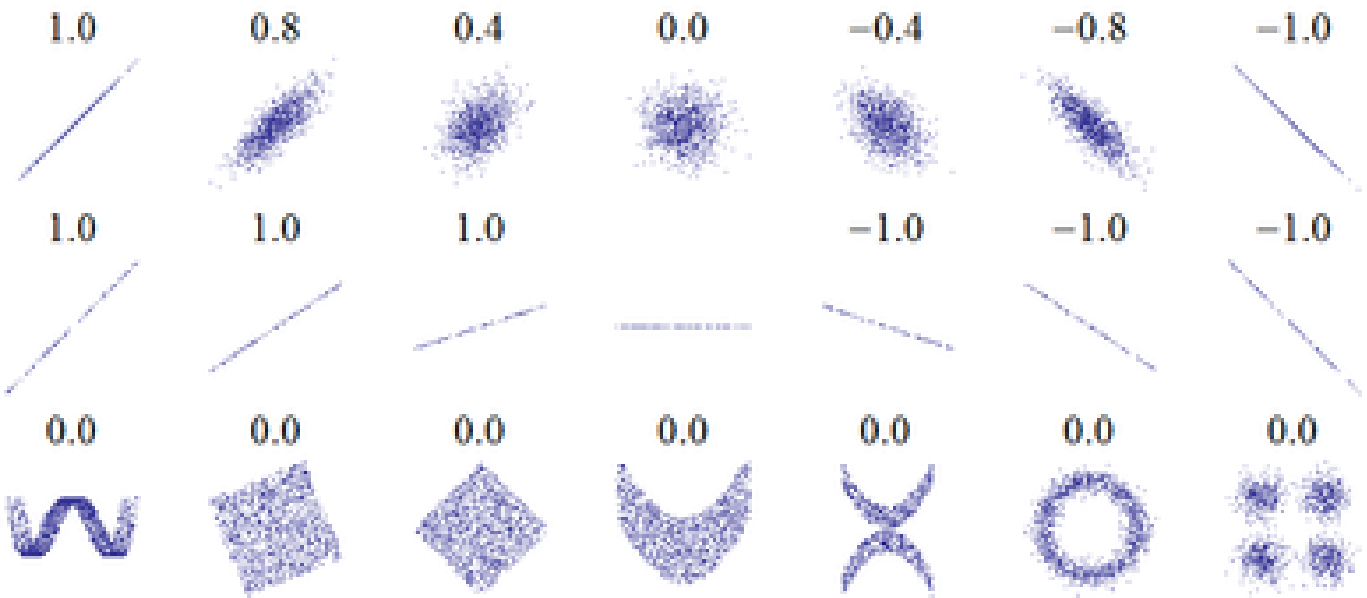
Henderson & Velleman (1981):
Building multiple regression models interactively



Theoretical background

Correlation

$$r_{xy} = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{(n-1)s_x s_y} = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^{n}(x_i - \bar{x})^2 \sum_{i=1}^{n}(y_i - \bar{y})^2}}$$



$$\hat{r}_{XY \cdot \mathbf{Z}} = \frac{N\sum_{i=1}^{N} r_{X,i} r_{Y,i} - \sum_{i=1}^{N} r_{X,i} \sum_{i=1}^{N} r_{Y,i}}{\sqrt{N\sum_{i=1}^{N} r_{X,i}^2 - \left(\sum_{i=1}^{N} r_{X,i}\right)^2} \sqrt{N\sum_{i=1}^{N} r_{Y,i}^2 - \left(\sum_{i=1}^{N} r_{Y,i}\right)^2}}$$

so for three variables:

$$\hat{r}_{XY \cdot \mathbf{Z}} = \frac{r_{XY} - r_{X\mathbf{Z}} r_{Y\mathbf{Z}}}{\sqrt{(1 - r_{X\mathbf{Z}}^2)(1 - r_{Y\mathbf{Z}}^2)}}$$

Exercises

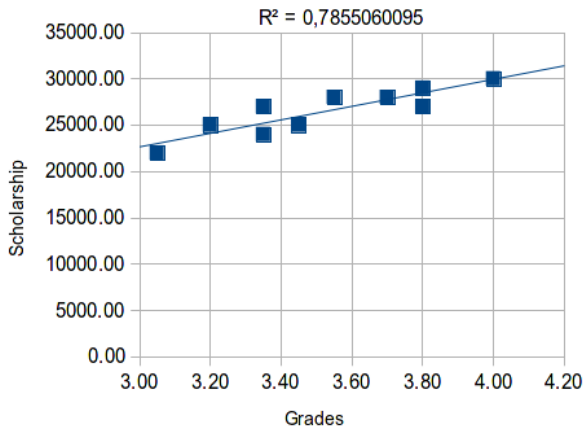
1. What is correlation and partial correlation?
2. Building upon your findings, compute the possible pairs of correlation coefficients on the below dataset!
3. Also look for partial correlation and comment on your results!

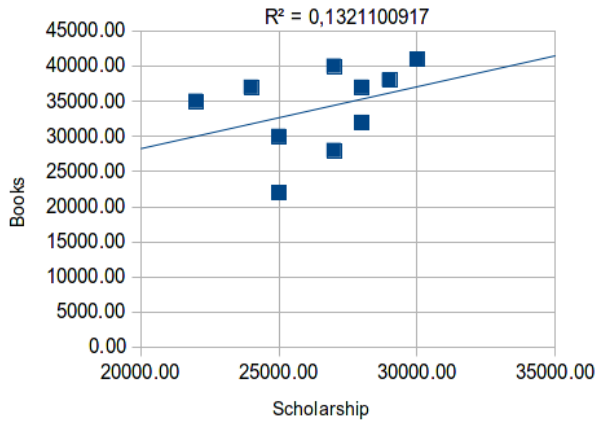
Grade (mean)	Scholarship (in HUF)	Money spent on books (in HUF)
3.05	22000	3500
3.2	25000	3000
3.35	27000	2800
3.35	24000	3700
3.45	25000	2200
3.55	28000	3200
3.7	28000	3700
45	30000	4100
3.8	27000	4000
3.8	29000	3800

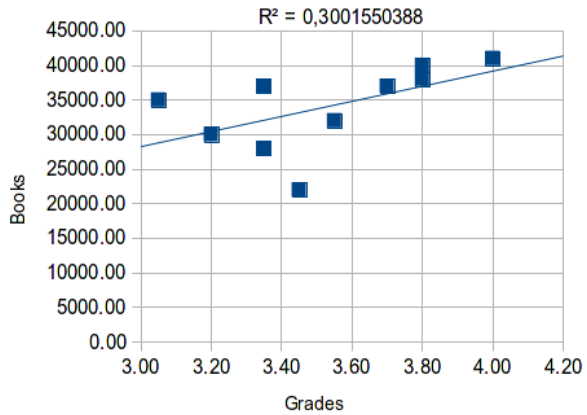
Exercises

Solution

				$x_i - \bar{x}$			$(x_i - \bar{x})^2$			$(x_i - \bar{x})(y_i - \bar{y})$		
	Grade	Scholarship	Books	Grade	Scholarship	Books	Grade	Scholarship	Books	Grade-sch	Sch-books	Grade-books
	3.05	22000.00	35000.00	-0.48	-4500.00	1000.00	0.225625	20250000	1000000	2137.5	-4500000	-475
	3.20	25000.00	30000.00	-0.33	-1500.00	-4000.00	0.105625	2250000	16000000	487.5	6000000	1300
	3.35	27000.00	28000.00	-0.18	500.00	-6000.00	0.030625	250000	36000000	-87.5	-3000000	1050
	3.35	24000.00	37000.00	-0.18	-2500.00	3000.00	0.030625	6250000	9000000	437.5	-7500000	-525
	3.45	25000.00	22000.00	-0.07	-1500.00	-12000.00	0.005625	2250000	144000000	112.5	18000000	900
	3.55	28000.00	32000.00	0.02	1500.00	-2000.00	0.000625	2250000	4000000	37.5	-3000000	-50
	3.70	28000.00	37000.00	0.18	1500.00	3000.00	0.030625	2250000	9000000	262.5	4500000	525
	4.00	30000.00	41000.00	0.48	3500.00	7000.00	0.225625	12250000	49000000	1662.5	24500000	3325
	3.80	27000.00	40000.00	0.28	500.00	6000.00	0.075625	250000	36000000	137.5	3000000	1650
	3.80	29000.00	38000.00	0.28	2500.00	4000.00	0.075625	6250000	16000000	687.5	10000000	1100
Min	3.05	22000.00	22000.00			Sum	0.80625	54500000	320000000	5875	48000000	8800
Max	4.00	30000.00	41000.00			St. dev.	0.29930475	2460.80384	5962.84794	r	0.89	0.36	0.55
Range	0.95	8000.00	19000.00							r^2	0.79	0.13	0.30
Mean	3.53	26500.00	34000.00							partial corr	0.96	-0.24	0.46
Median	3.50	27000.00	36000.00										





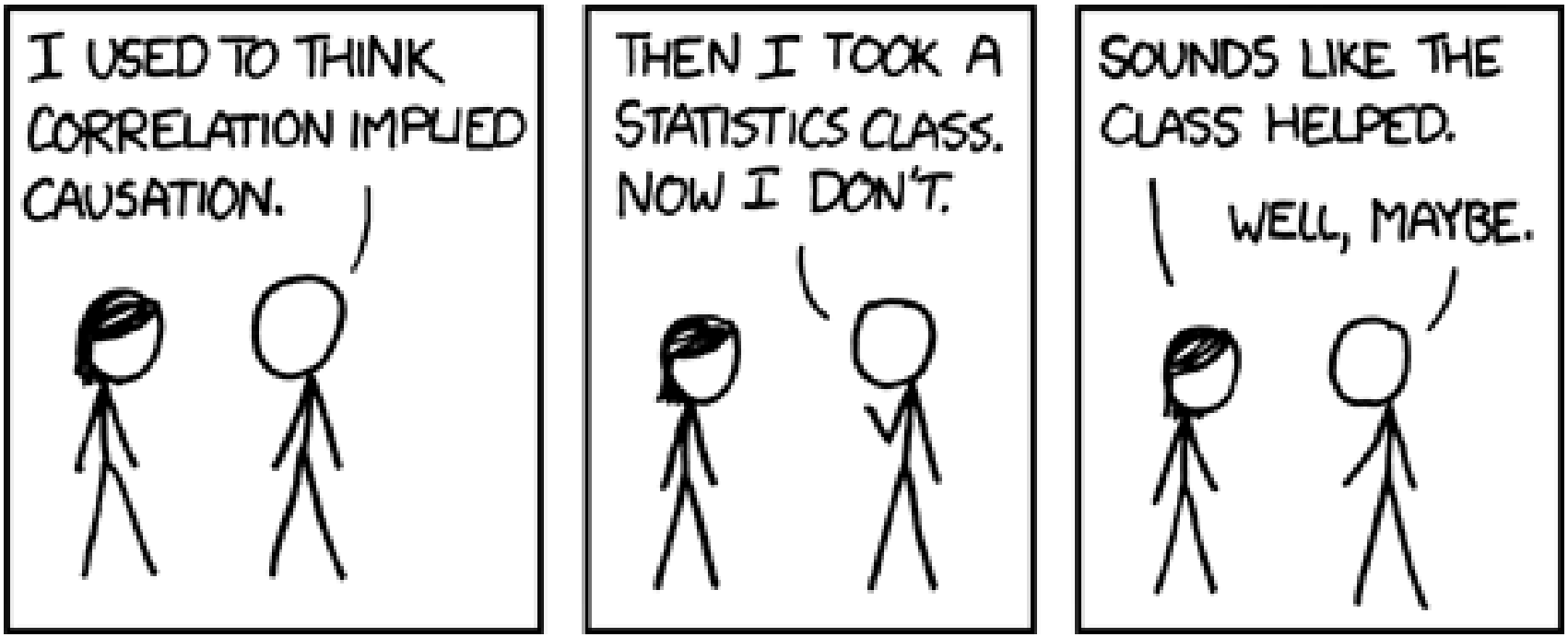


Limitations of the correlation coefficient

- Correlation and causality
- Lazarsfeld paradigm
- Correlation and linearity

Limitations of the correlation coefficient

Correlation does not imply causation!



Source: http://xkcd.com/552

Limitations of the correlation coefficient

Correlation does not imply causation! - Theoretical background

Aristotle: logic, syllogism – if $(A \rightarrow B)\&(B \rightarrow C) \Rightarrow A \rightarrow C$

David Hume: scepticism

- "only correlation can actually be perceived [not causality]"
- see: our belief that the sun will rise tomorrow
- see: "If I see a billiard ball moving towards another, on a smooth table, I can easily conceive to stop upon contact."

Popper: falsification

Pearl, J. - *Causality: Models, Reasoning, and Inference*, Cambridge University Press, 2000

Stouffer: *The American Soldier*

Soldiers in branches with higher promotion rates are happier than soldiers in branches with lower rates of promotion.

Stouffer: *The American Soldier*

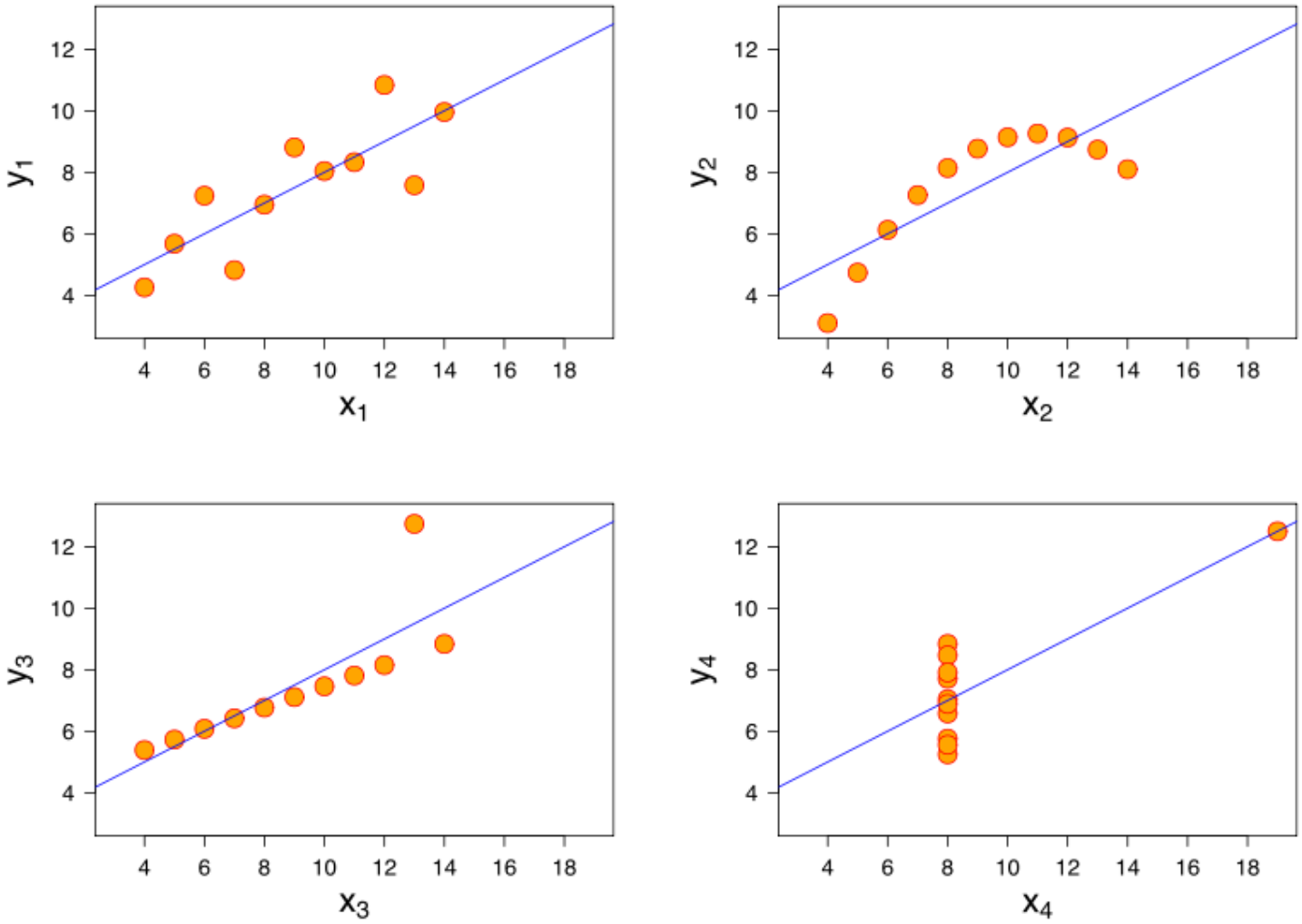
H_0: Soldiers in branches with higher promotion rates are happier than soldiers in branches with lower rates of promotion. **BUT:**

"Soldiers in branches with higher promotion rates were more pessimistic about their own chances of being promoted than soldiers in branches with lower rates of promotion."

Keywords: **reference group**, **relative deprivation**

Limitations of the correlation coefficient

Correlation and linearity - Variations of the Same Theme



Source: Anscombe, F. J. (1973) Graphs in statistical analysis. American Statistician, 27, 17–21.

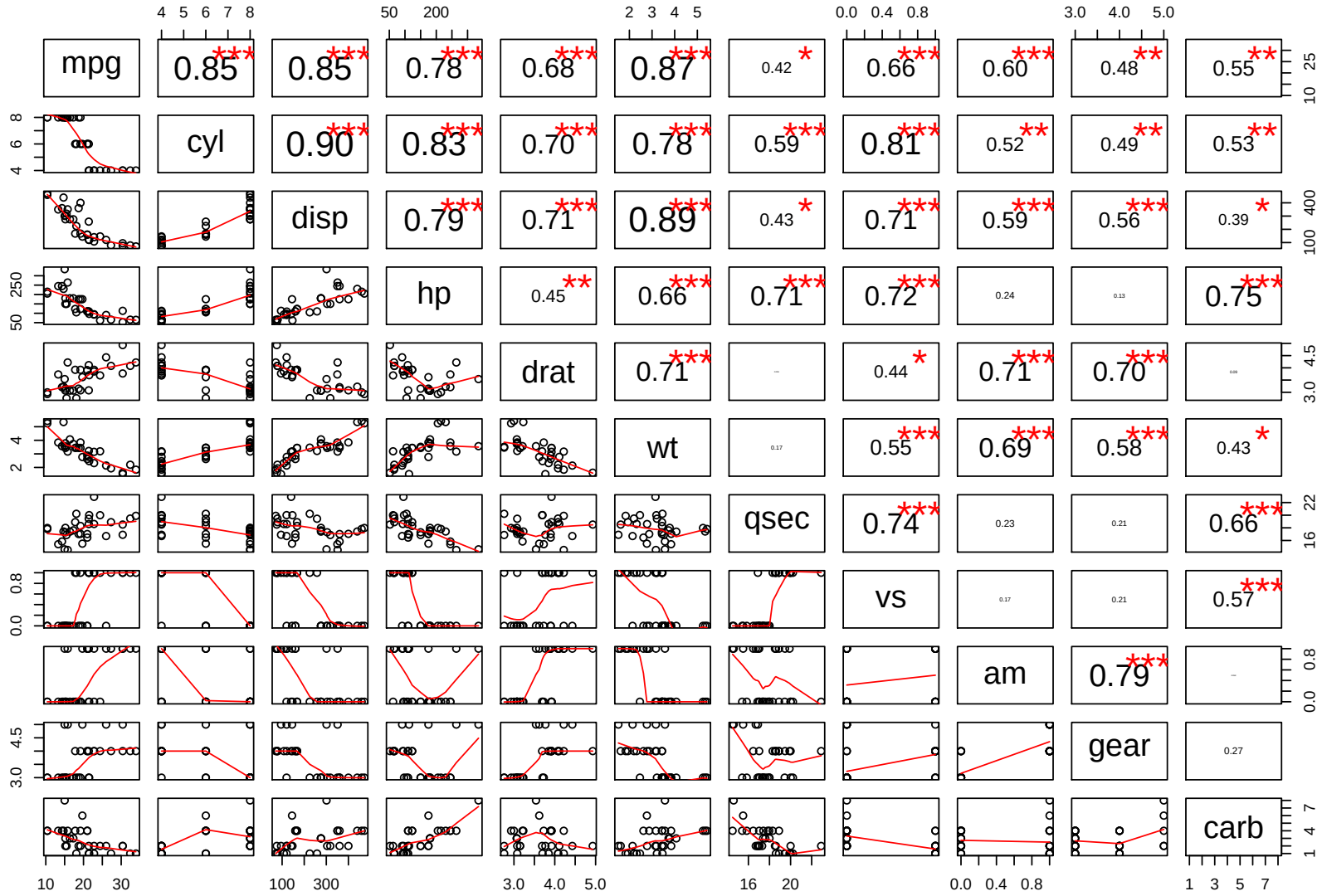
Exercise

The data was extracted from the 1974 Motor Trend US magazine, and comprises fuel consumption and 10 aspects of automobile design and performance for 32 automobiles (1973-74 models).

- mpg: Miles/(US) gallon
- cyl: Number of cylinders
- disp: Displacement (cu.in.)
- hp: Gross horsepower
- drat: Rear axle ratio
- wt: Weight (lb/1000)
- qsec: 1/4 mile time
- vs: V/S
- am: Transmission (0 = automatic, 1 = manual)
- gear: Number of forward gears
- carb: Number of carburetors

Source: Henderson and Velleman (1981), Building multiple regression models interactively. *Biometrics*, 37, 391-411.

Exercise



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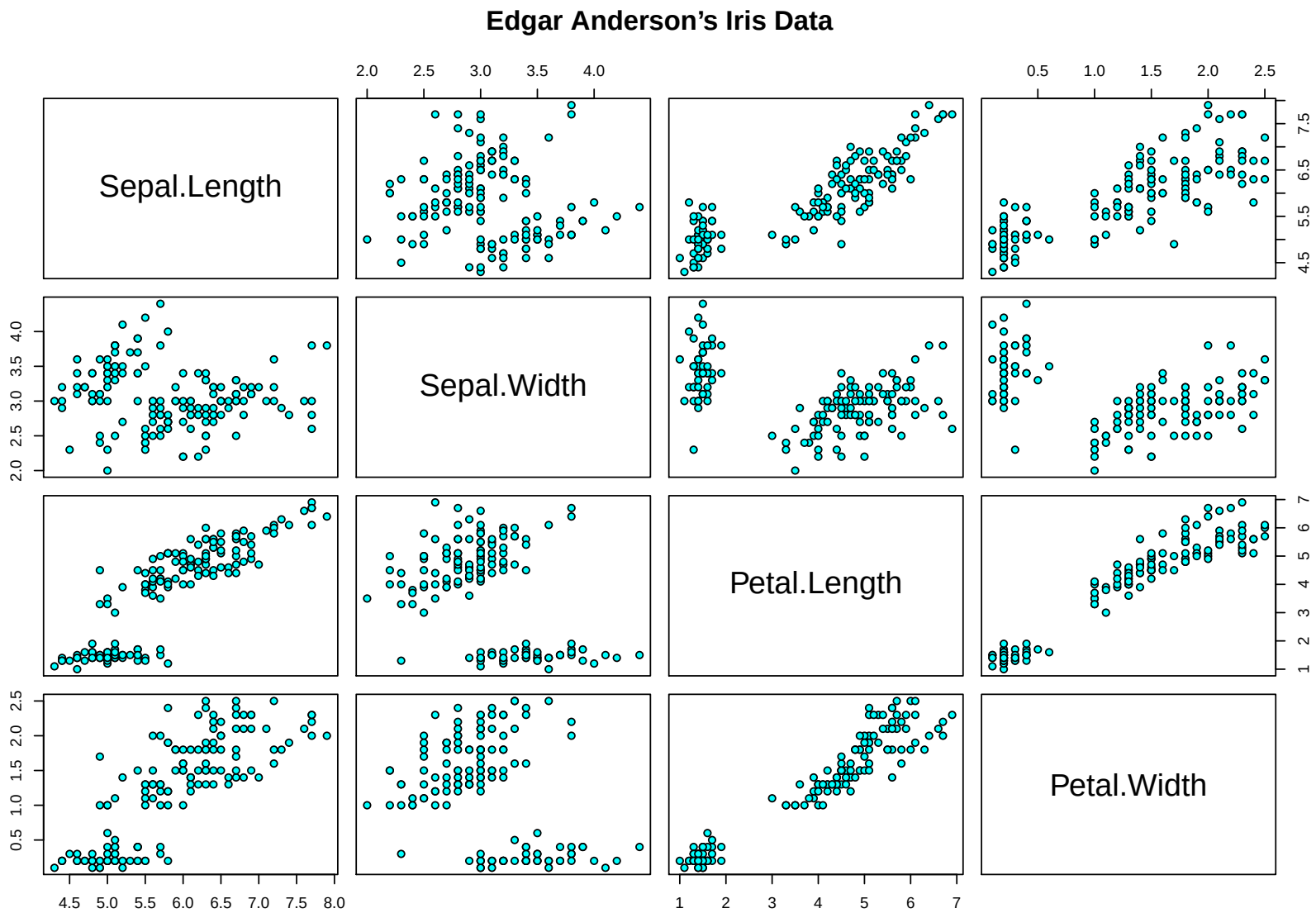
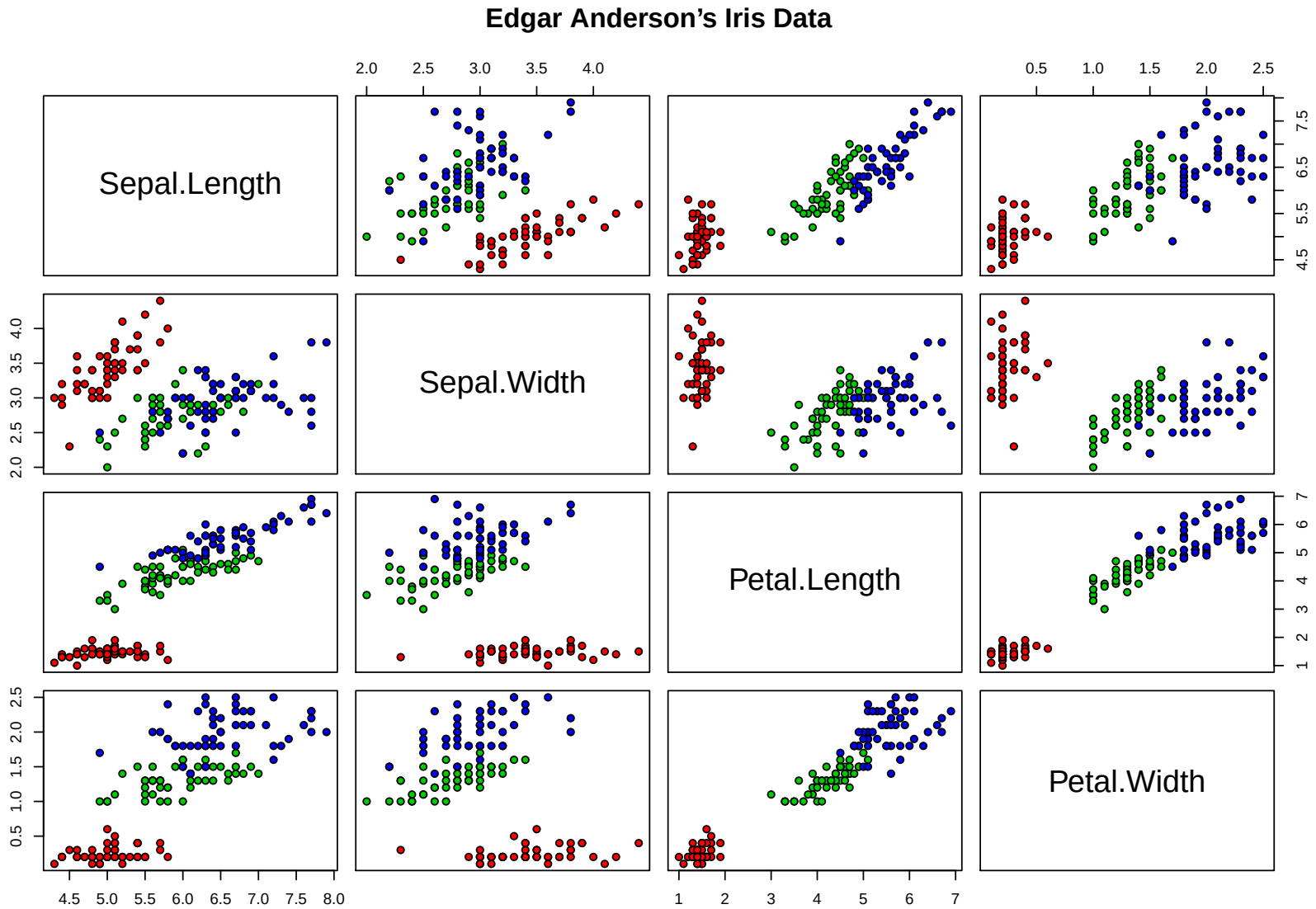
Exercise



Anderson, Edgar (1935). The irises of the Gaspe Peninsula, *Bulletin of the American Iris Society*, 59, 2-5.

Exercise #2



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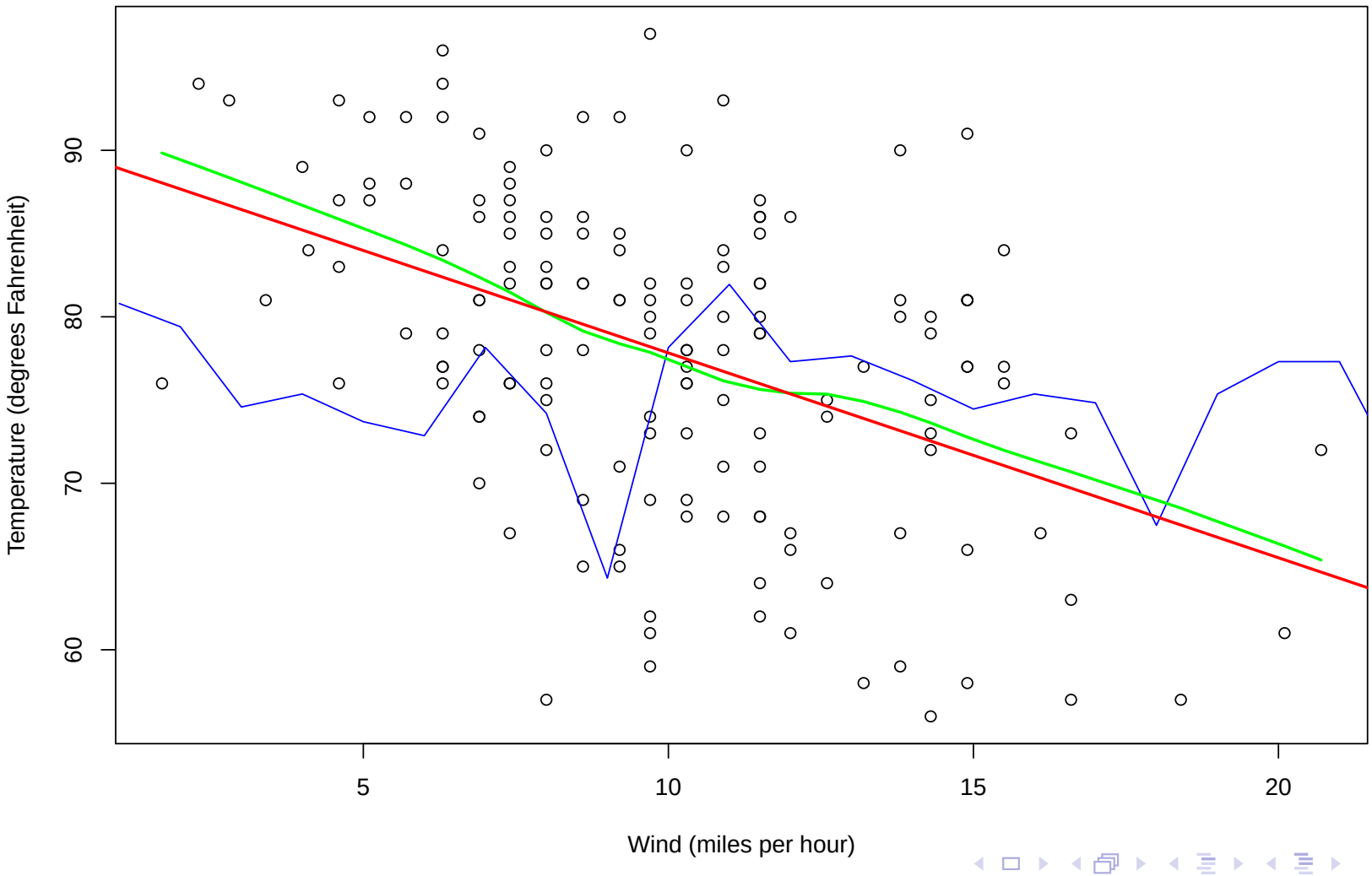
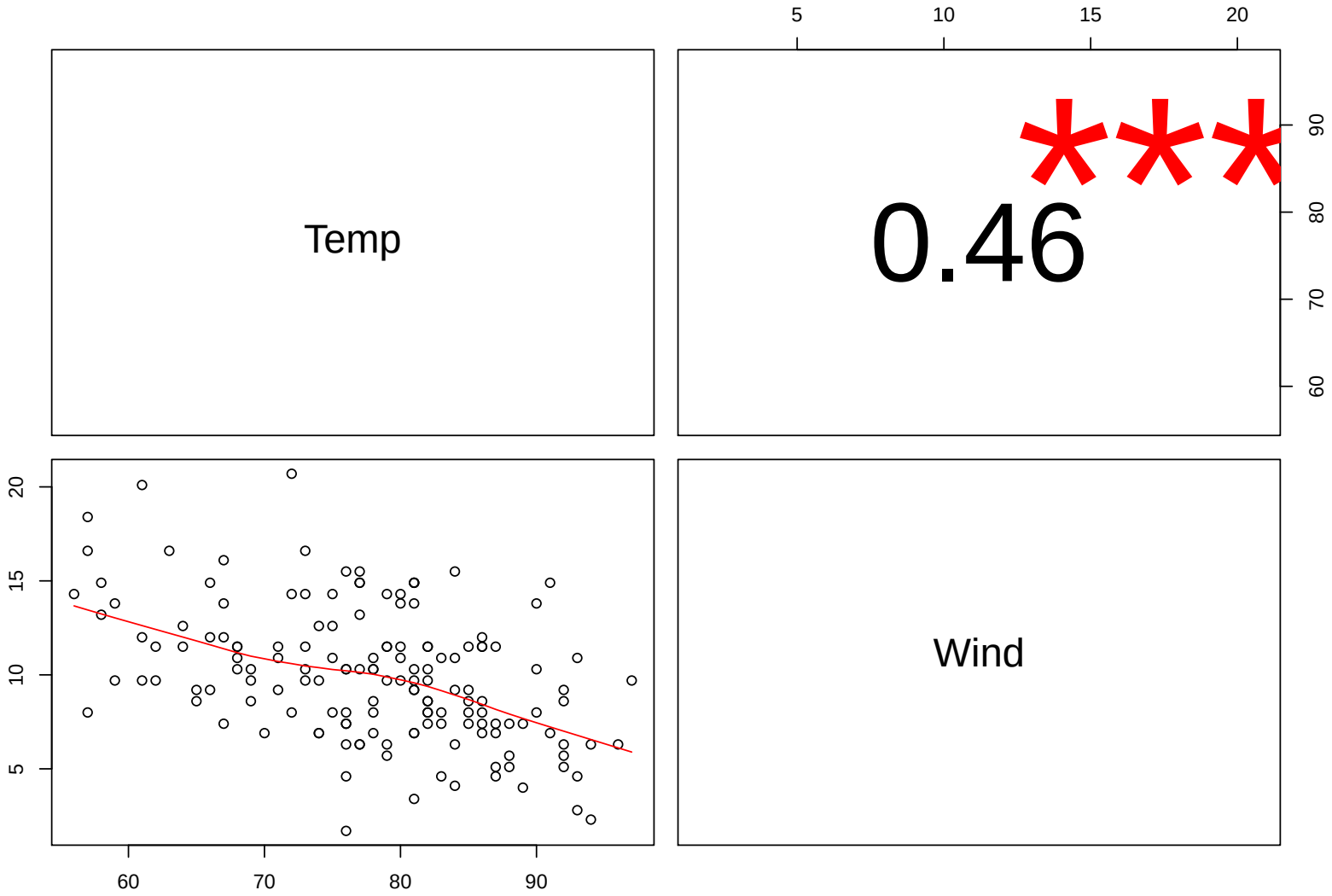
Exercise #3

Real association?



Exercise #3

Real association?



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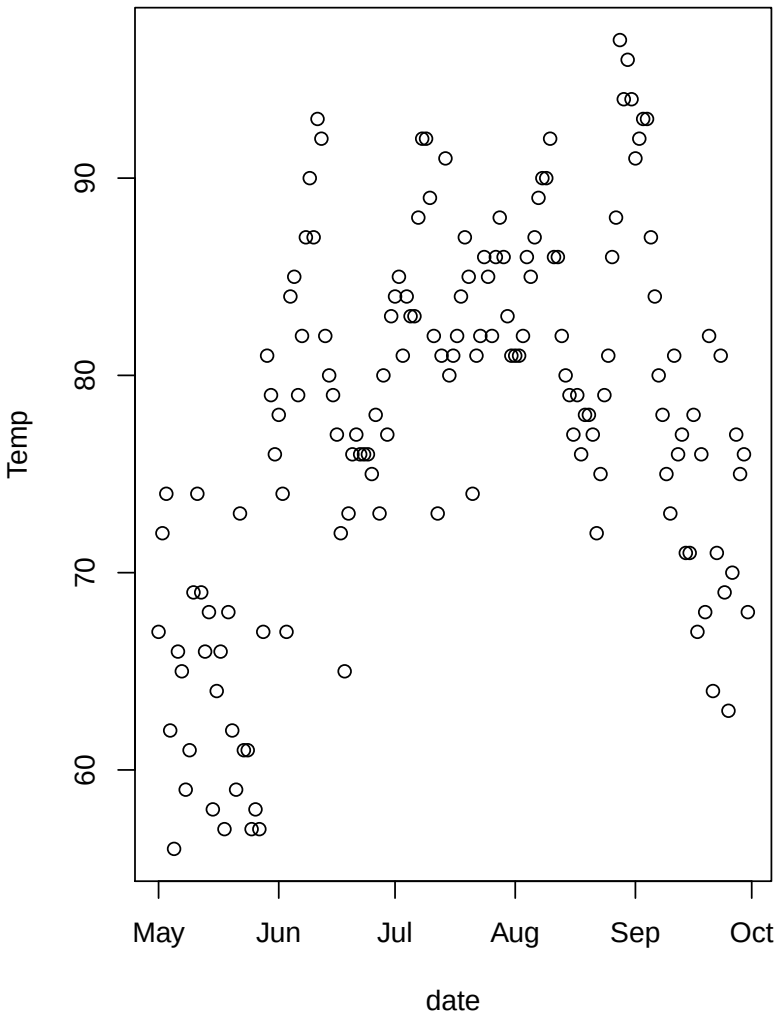
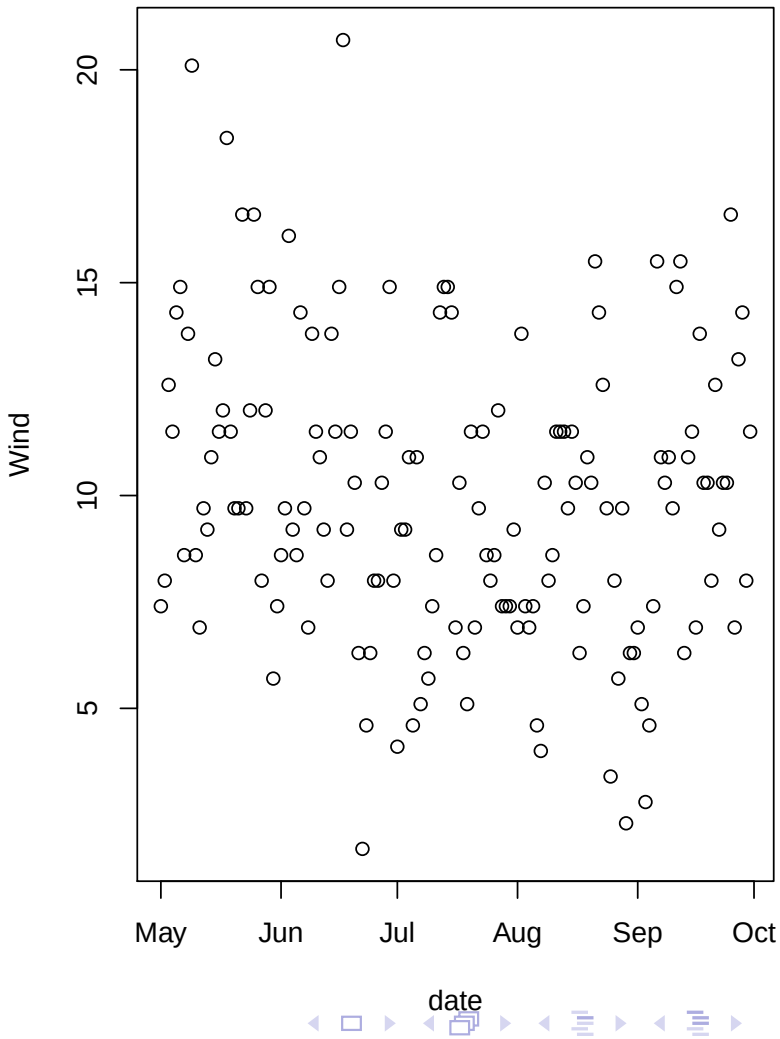
Real association?





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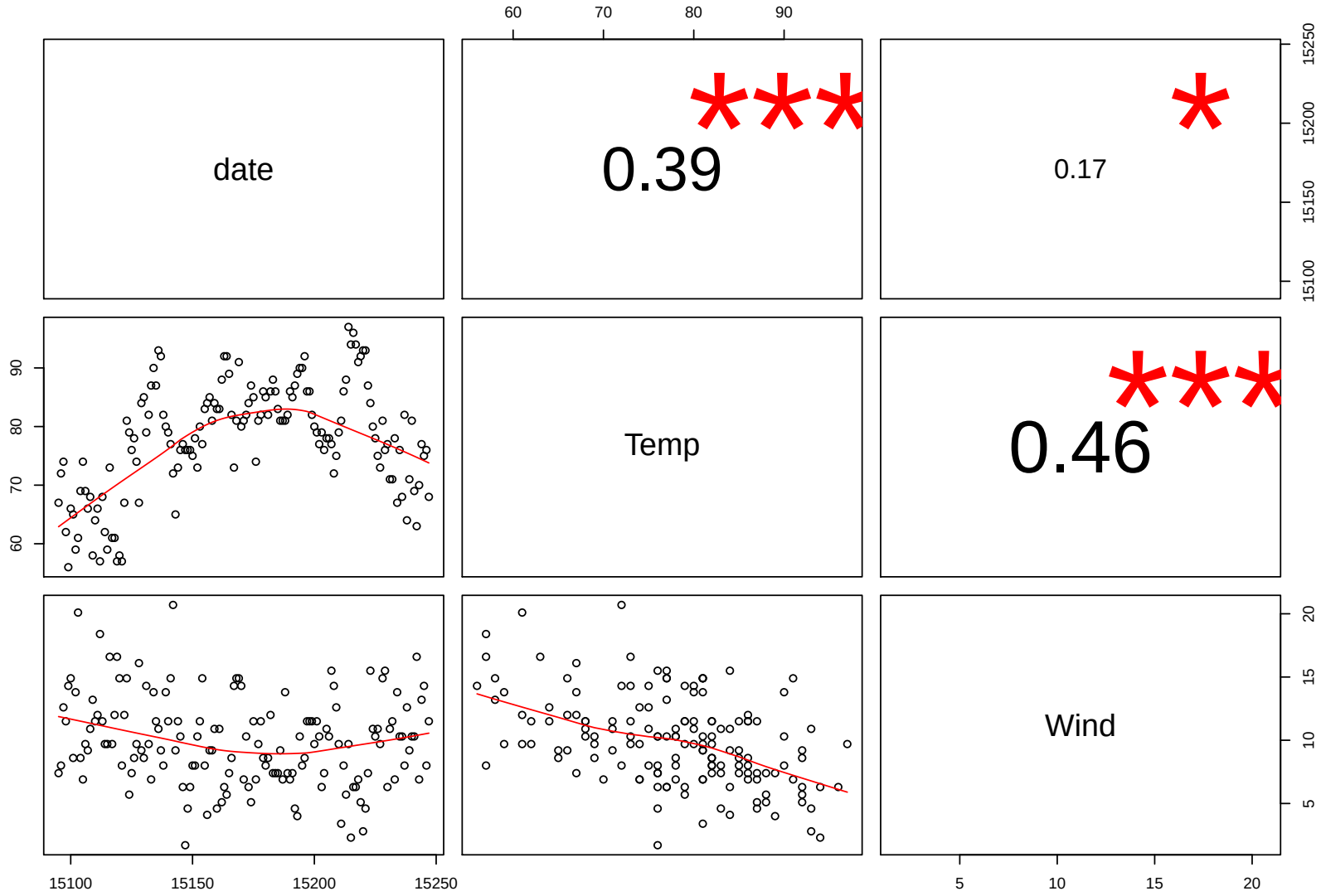
Real association?



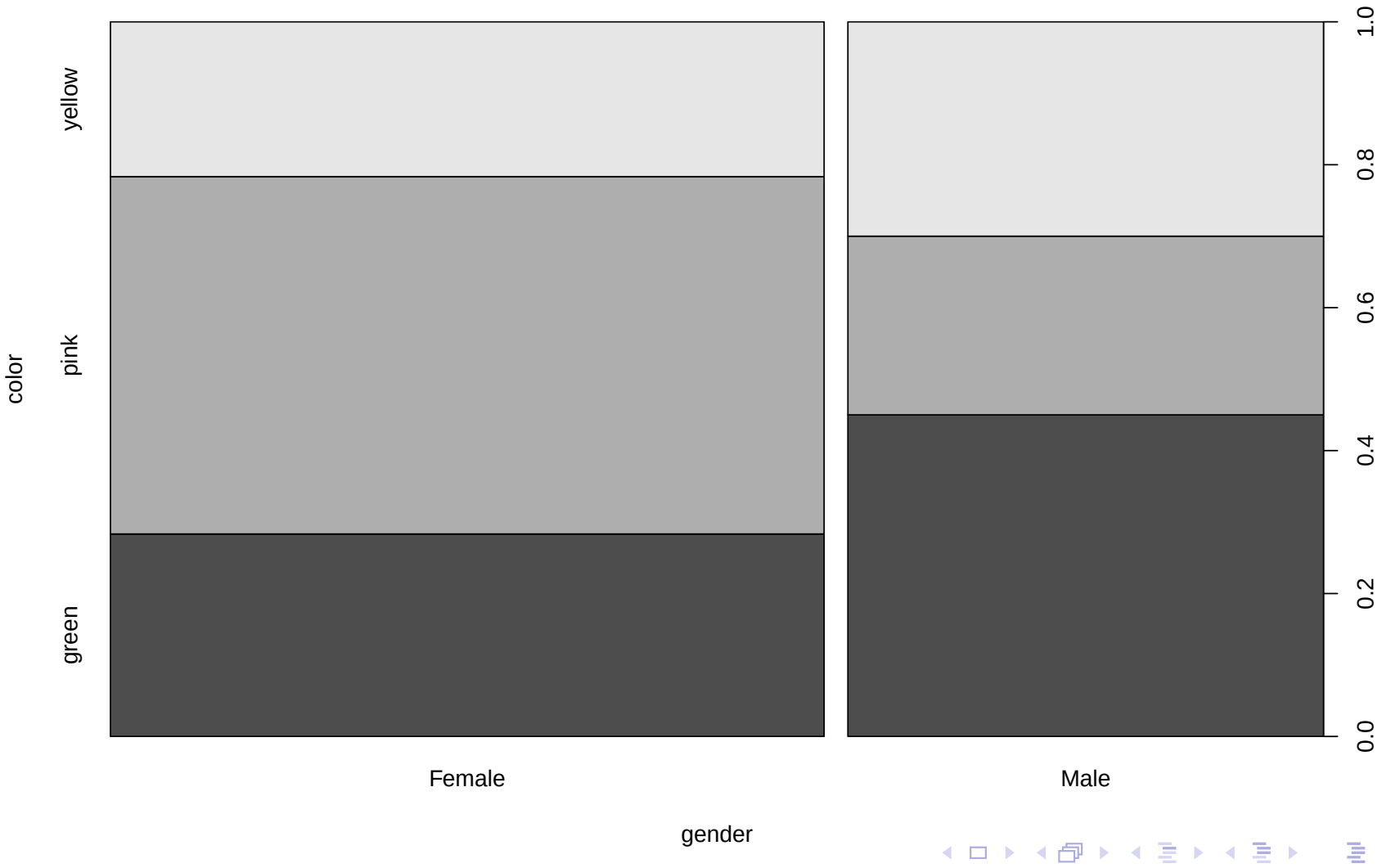
Crosstables

Discrete (qualitative) variables

ID	gender	color
1	Female	pink
2	Female	pink
3	Female	pink
4	Female	pink
5	Female	pink
6	Female	pink
	...	
95	Male	yellow
96	Male	yellow
97	Male	yellow
98	Male	yellow
99	Male	yellow
100	Male	yellow

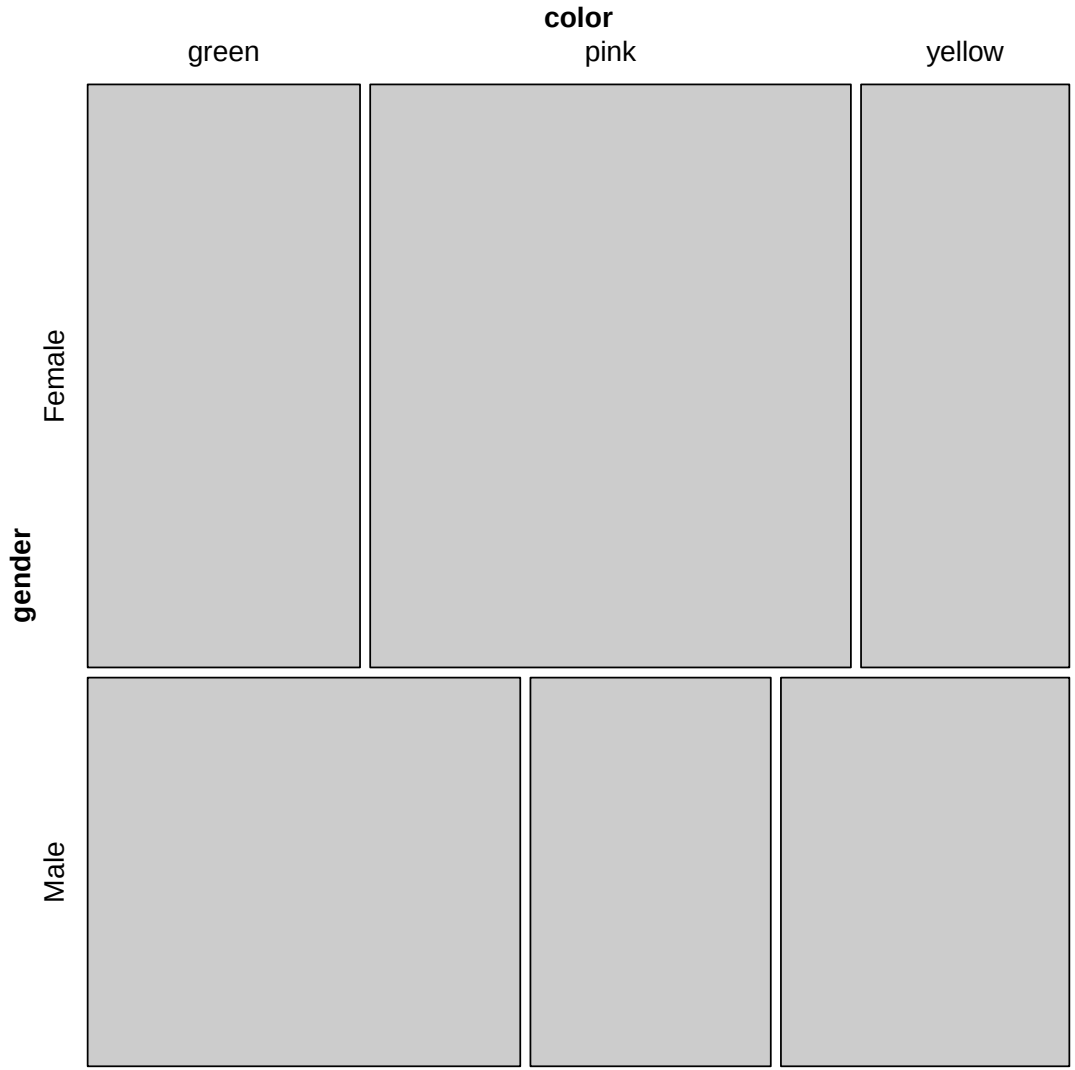
Crosstables

Discrete (qualitative) variables



Crosstables

Discrete (qualitative) variables



Crosstables

Discrete (qualitative) variables

	green	pink	yellow
Female	17	30	13
Male	18	10	12

Crosstables

Discrete (qualitative) variables

	green	pink	yellow	
Female	17	30	13	Marginals
Male	18	10	12	
	Marginals			N

Crosstables

Discrete (qualitative) variables

	green	pink	yellow	$\sum$
Female	17	30	13	60
Male	18	10	12	40
$\sum$	35	40	25	100

Crosstables

Percentages

	green	pink	yellow	$\sum$
Female	17	30	13	60
Male	18	10	12	40
$\sum$	35	40	25	100

Table: Counted values

	green	pink	yellow	$\sum$
Female	17 %	30 %	13 %	60 %
Male	18 %	10 %	12 %	40 %
$\sum$	35 %	40 %	25 %	100 %

Table: Total percentages

Crosstables

Row percentages

	green	pink	yellow	$\sum$
Female	17	30	13	60
Male	18	10	12	40
$\sum$	35	40	25	100

Table: Counted values

	green	pink	yellow	$\sum$
Female	28.3 %	50 %	21.7 %	100 %
Male	45 %	25 %	30 %	100 %
$\sum$	35 %	40 %	25 %	100 %

Table: Row percentages

Crosstables

Column percentages

	green	pink	yellow	$\sum$
Female	17	30	13	60
Male	18	10	12	40
$\sum$	35	40	25	100

Table: Counted values

	green	pink	yellow	$\sum$
Female	48.63 %	75 %	52 %	60 %
Male	51.4 %	25 %	48 %	40 %
$\sum$	100 %	100 %	100 %	100 %

Table: Column percentages

Crosstables

Expected values

	green	pink	yellow	$\sum$
Female	17	30	13	60
Male	18	10	12	40
$\sum$	35	40	25	100

Table: Counted values

	green	pink	yellow	$\sum$
Female	21	24	15	60
Male	14	16	10	40
$\sum$	35	40	25	100

Table: Expected values

$$\chi^2 = \sum_{i=1}^{n} \frac{(O_i - E_i)^2}{E_i}$$

where:

- χ^2: Pearson's cumulative test statistic,
- O_i: an observed (counted) frequency,
- E_i: an expected (theoretical) frequency,
- n: the number of cells in the table.

H_0: observed and expected values are all the same

Requirements!

Crosstables

Computed chi-square

	green	pink	yellow	$\sum$
Female	$\frac{(17-21)^2}{21}$	$\frac{(30-24)^2}{24}$	$\frac{(13-15)^2}{15}$	-
Male	$\frac{(18-14)^2}{14}$	$\frac{(10-16)^2}{16}$	$\frac{(12-10)^2}{10}$	-
$\sum$	-	-	-	-

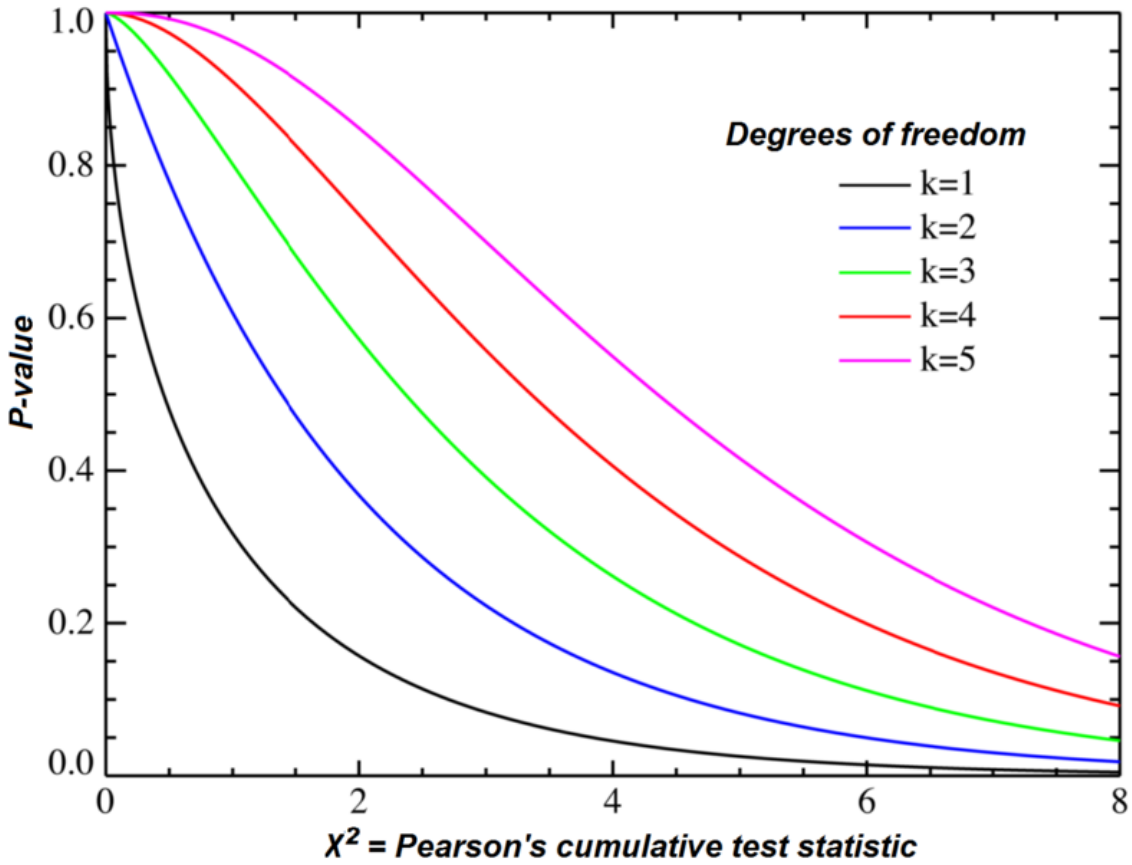
Table: Computed distances between observed and expected values

$$\chi^2 = \sum_{i=1}^{n} \frac{(O_i - E_i)^2}{E_i} = 6.321429$$

degrees of freedom: $(3-1)(2-1) = 2$

Crosstables

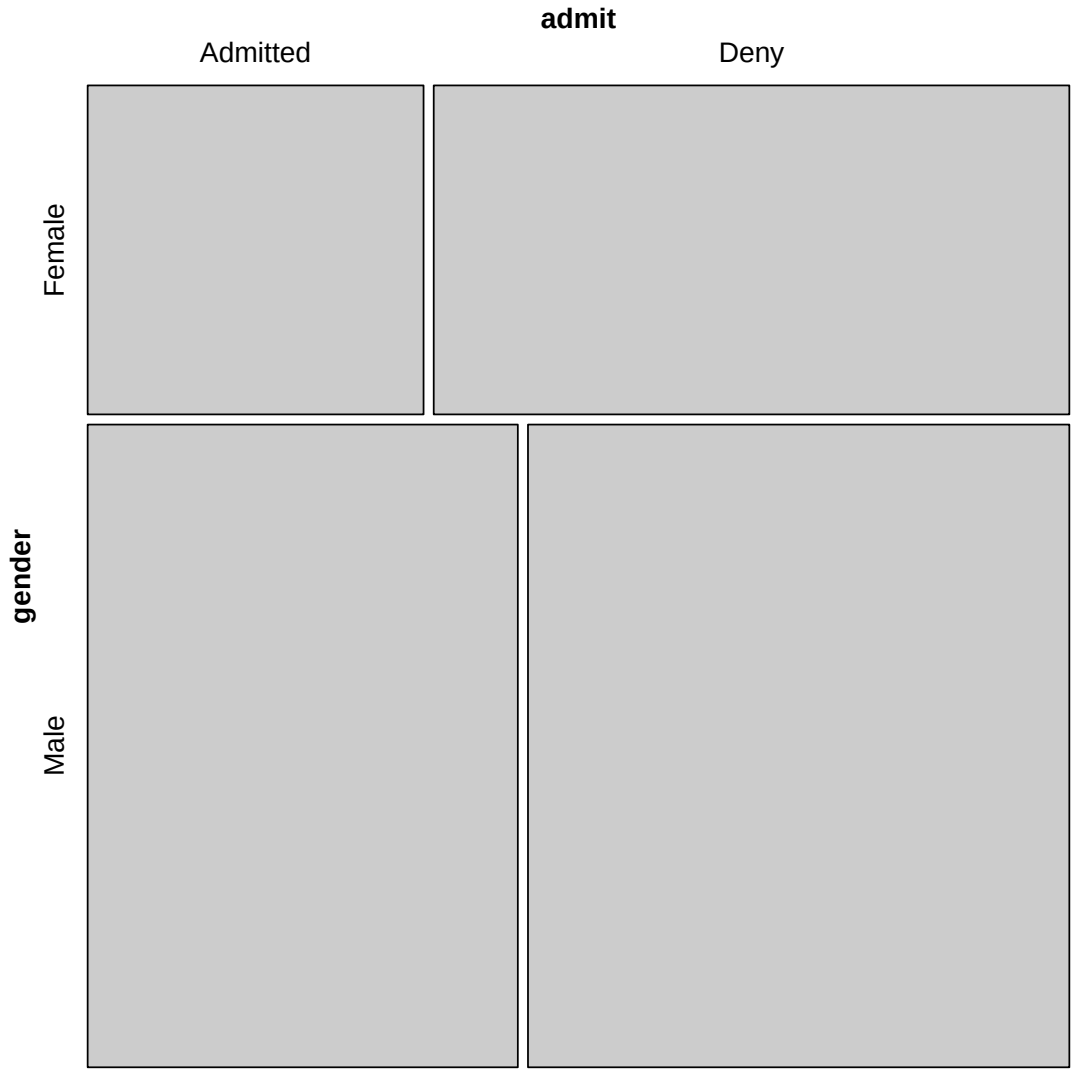
Computed chi-square



$$\Rightarrow p = 0.04239545$$

Simpson's paradox

Berkeley sex bias case



Simpson's paradox

Berkeley sex bias case

	Admitted	Deny	$\sum$
Female	1494	2827	4321
Male	3738	4704	8442
$\sum$	5232	7531	12763

Table: Observed values

	Admitted	Deny	$\sum$
Female	34.6 %	65.4 %	100 %
Male	44.3 %	55.7 %	100 %
$\sum$	41 %	59 %	100 %

Table: Row percentages

$$\chi^2 = 110.8489; d.f. = 1; p = 6.385628e-26$$

Simpson's paradox

Berkeley sex bias case

	Applicants	Admitted
Men	8442	**44%**
Women	4321	35%

	Men		Women	
Departement	Applicants	Admitted	Applicants	Admitted
A	825	62%	108	**82%**
B	560	63%	25	**68%**
C	325	**37%**	593	34%
D	417	33%	375	**35%**
E	191	**28%**	393	24%
F	272	6%	341	**7%**

Simpson's paradox

Batting averages in professional baseball

	1995		1996		Combined	
	Runs/Outs	%	Runs/Outs	%	Runs/Outs	%
Derek Jeter	12/48	25 %	183/582	31.4 %	195/630	**31** %
David Justice	104/411	**25.3** %	45/140	**32.1** %	149/551	27 %

Who is the better player?

Standardization and decomposition

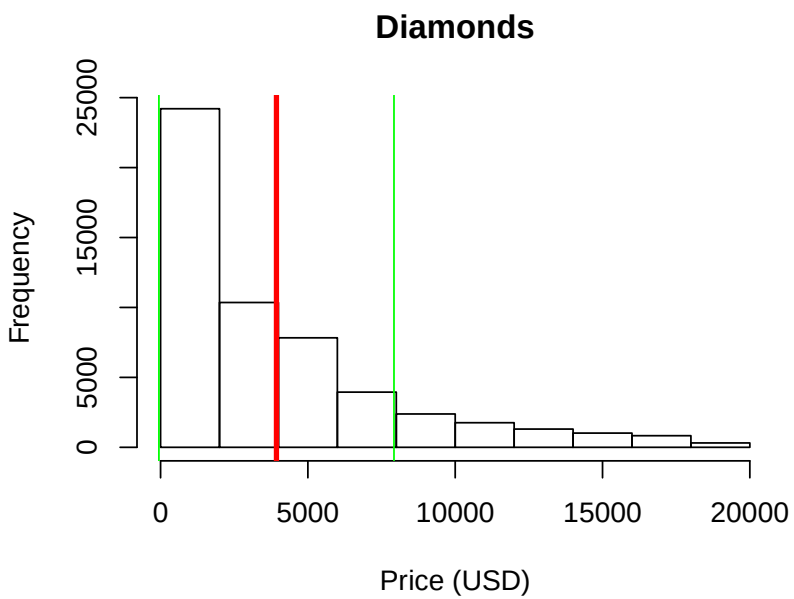
A basic example

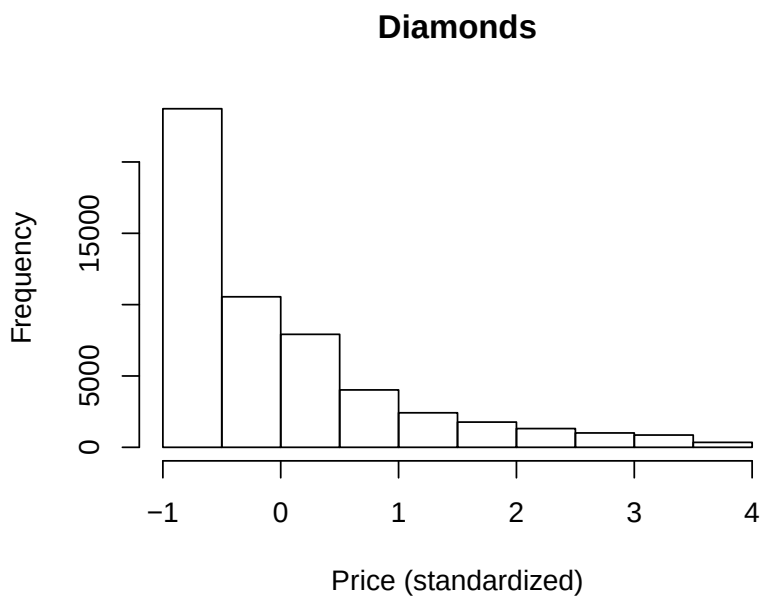
Standardization and decomposition

A basic example

Standard score (z-values, z-scores, normal scores, standardized variables) indicates how many standard deviations an observation is above or below the mean:

$$z = \frac{x - \mu}{\sigma}$$





Population and Deaths by Age in 1970 for White Females in Miami, Alaska, and the U.S.

	Miami			Alaska			U.S.		
Age	Pop.	Deaths	Rate*	Pop.	Deaths	Rate*	Pop.+	Deaths+	Rate*
< 15	114,350	136	1.19	37,164	59	1.59	23,961	32	1.34
15-24	80,259	57	0.71	20,036	18	0.90	15,420	9	0.58
25-44	133,440	208	1.56	32,693	37	1.13	21,353	30	1.40
45-64	142,670	1,016	7.12	14,947	90	6.02	19,609	140	7.14
65+	92,168	3,605	39.11	2,077	81	39.00	10,685	529	49.51
	562,887	5,022		106,917	285		91,028	740	
Crude death rate*			8.92			2.67			8.13

* Deaths per 1,000 population + in thousands

Definition

In direct standardization the stratum-specific rates of study populations are applied to the age distribution of a standard population.

$$Directly\ standardized\ rate = \frac{\sum stratum\ specific\ rates \times standard\ weights}{\sum standard\ weights}$$

$$Miami = \frac{(1.19x23,961) + \cdots + (39.11x10,685)}{91,208} = 6.92\ deaths/thousand$$

$$Alaska = \frac{(1.59x23,961) + \cdots + (39x10,685)}{91,208} = 6.71\ deaths/thousand$$

Definition

In indirect standardization, the standard population provides the rates and the study population provides the weights.

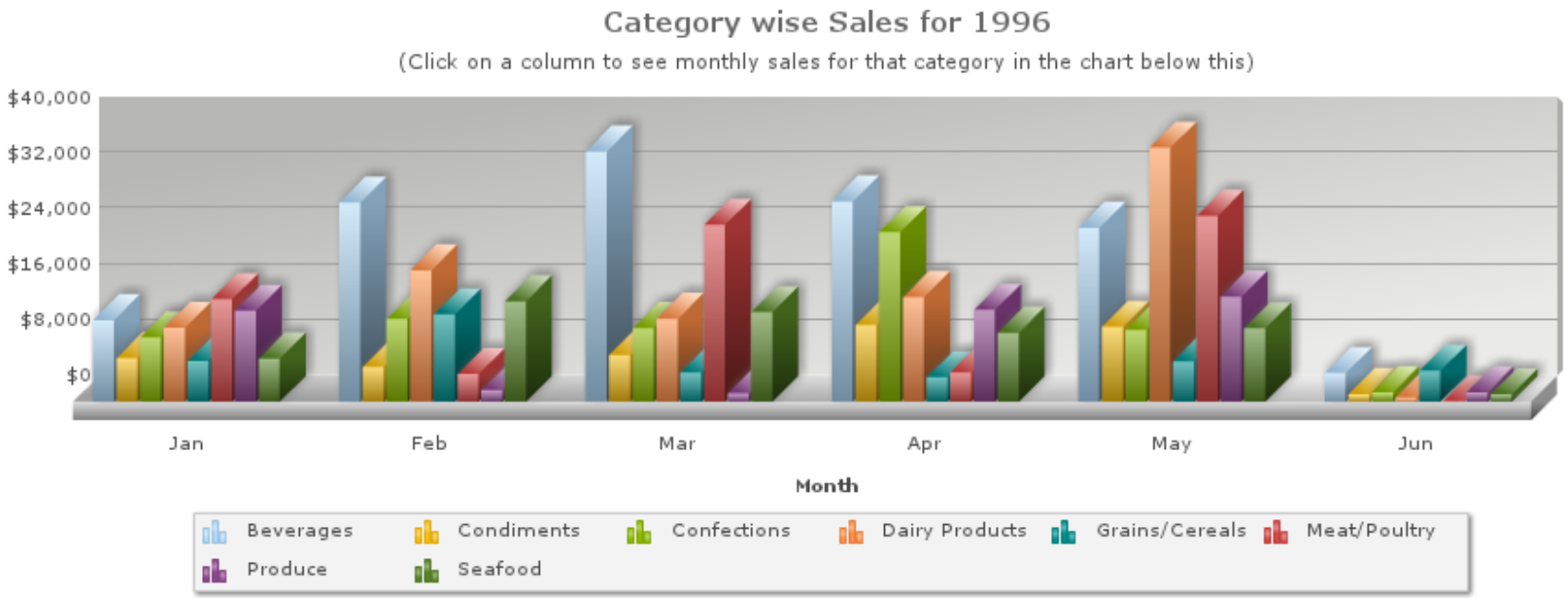
$$\textit{Indirectly standardized rate} = \frac{\sum \textit{observed values}}{\sum \textit{expected values}}$$

Expected values = Stratum specific rates from the study population $\times$ stratum sizes from the study population

	Study population	**Standard population**
Directly-standardized rate	Rates	Weights
Indirectly-standardized rate	Weights	Rates

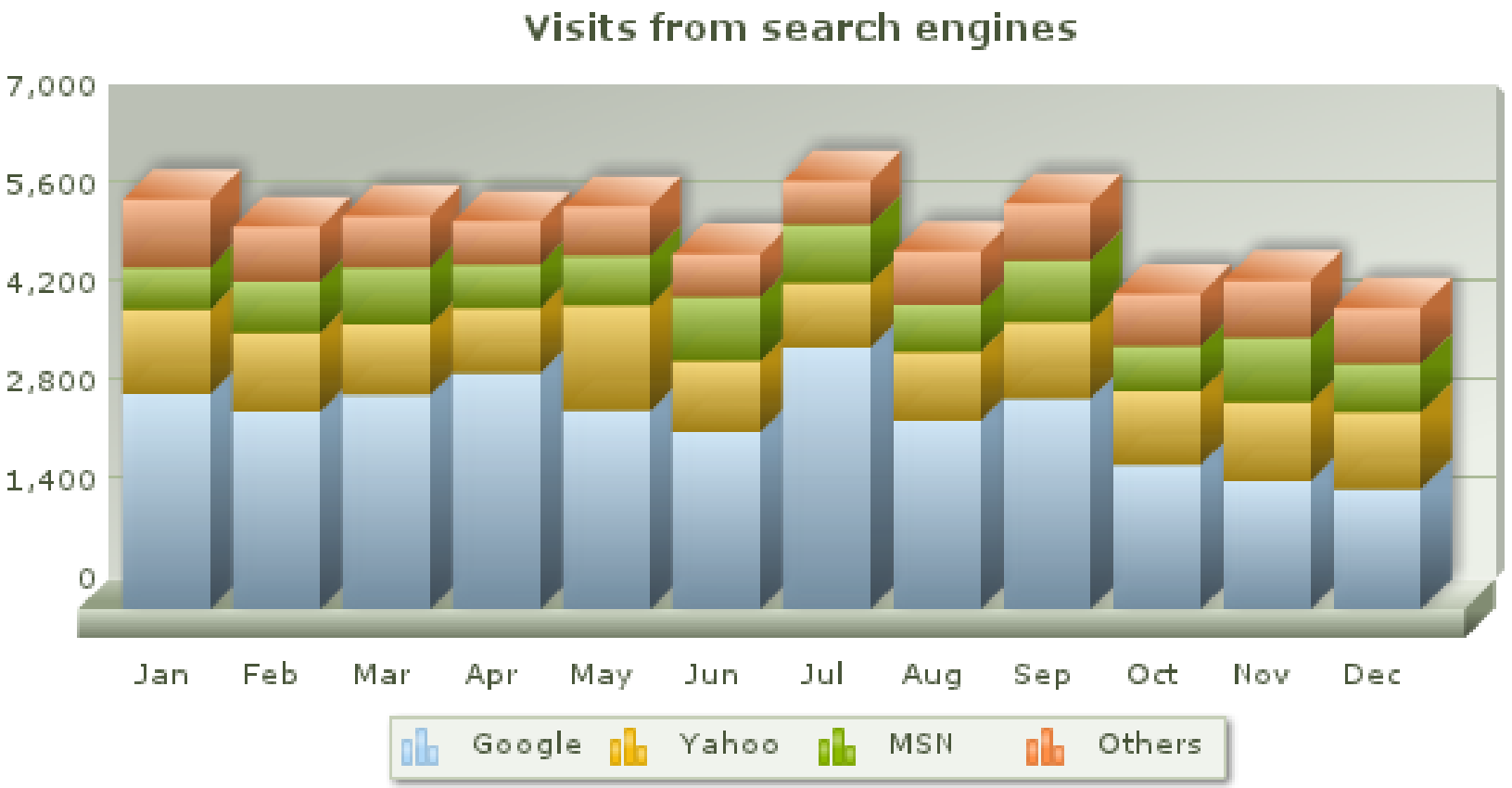
Graphs

Dodged bar



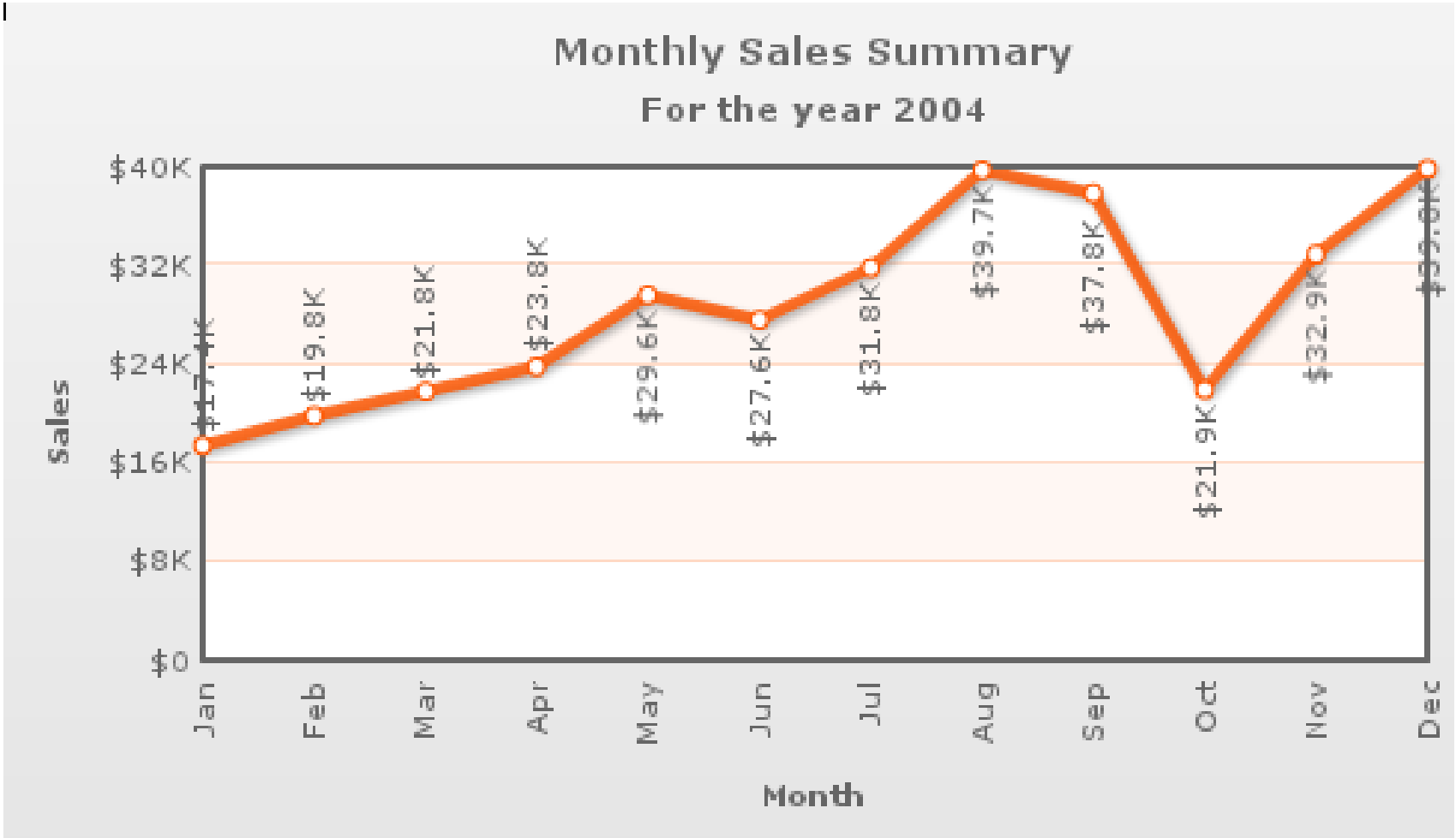
Graphs

Stacked bar



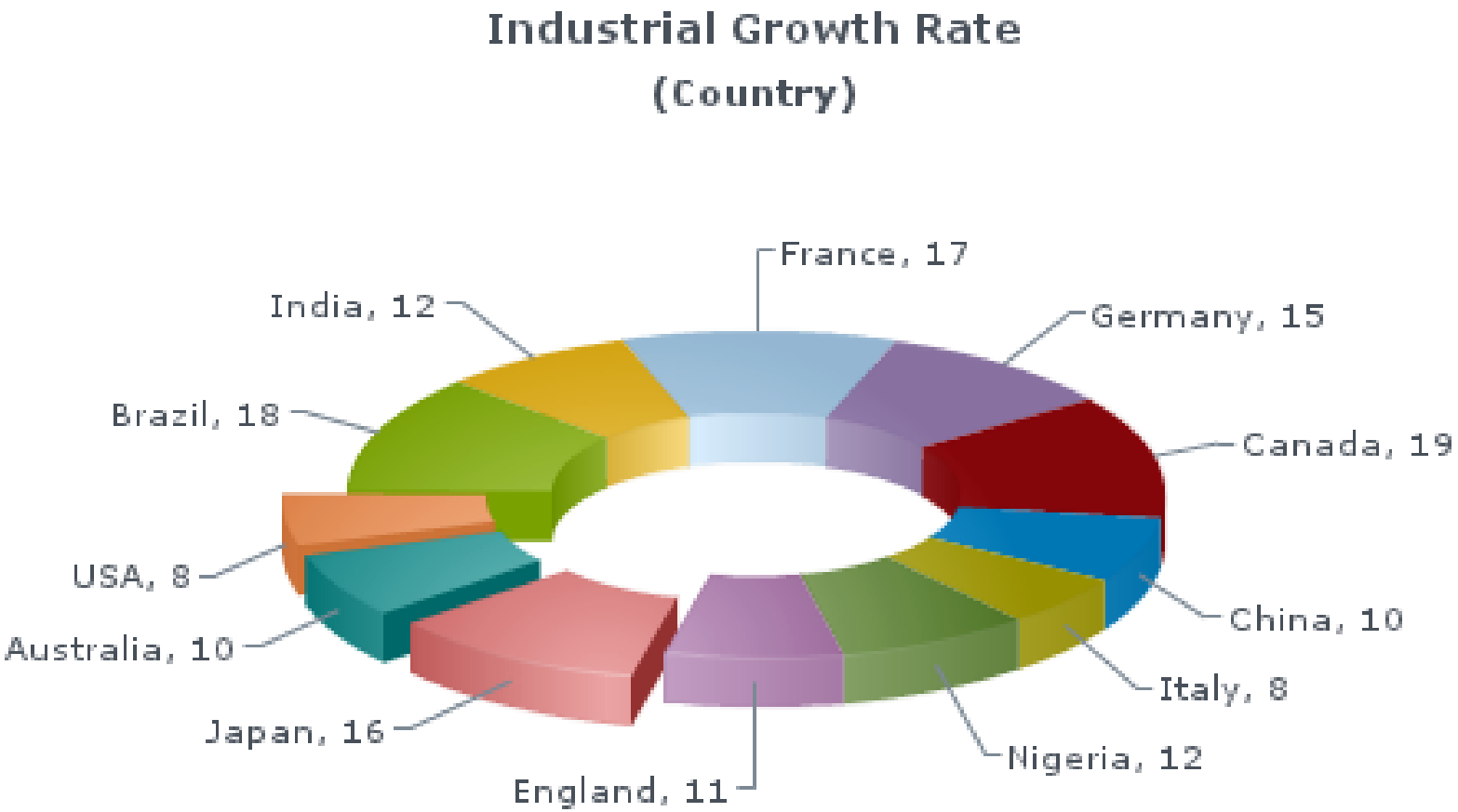
Graphs

Line


Monthly Sales Summary
For the year 2004
$40K
$32K
$24K
$16K
$8K
$0
Sales
$19.8K
$21.8K
$23.8K
$29.6K
$27.6K
$31.8K
$39.7K
$37.8K
$21.9K
$32.9K
Jan
Feb
Mar
Apr
May
Jun
Jul
Aug
Sep
Oct
Nov
Dec
Month

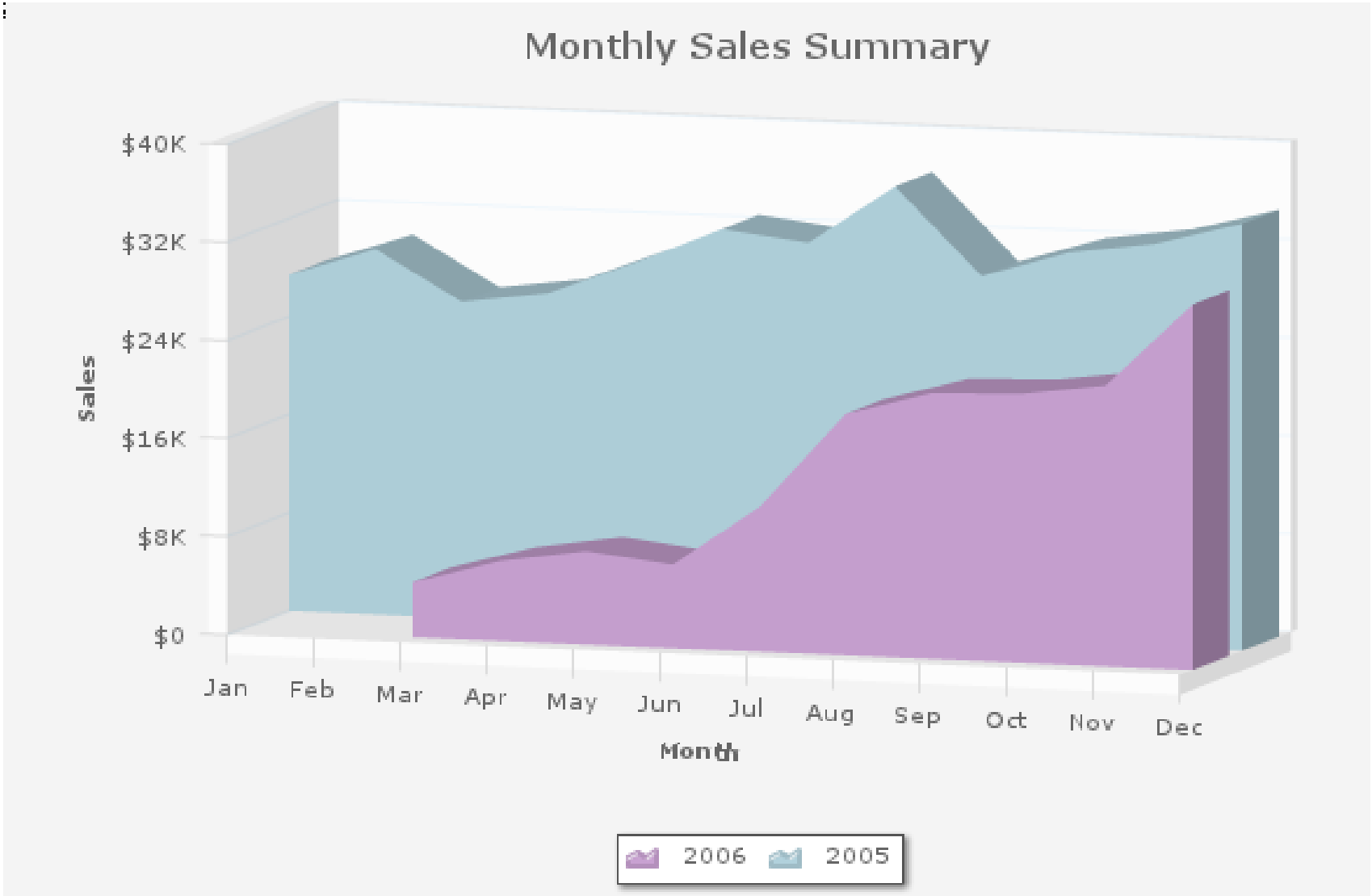
Graphs

Pie/donut



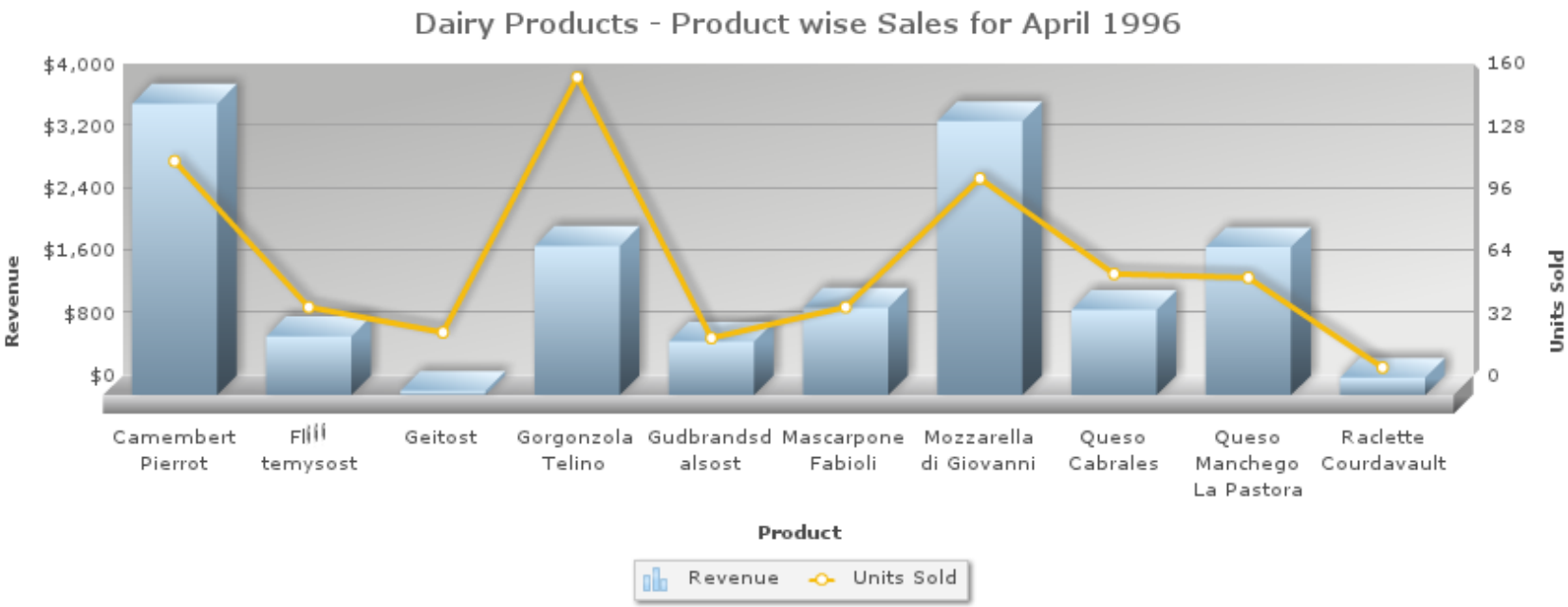
Graphs

Area


Monthly Sales Summary
$40K
$32K
$24K
$16K
$8K
$0
Sales
Jan
Feb
Mar
Apr
May
Jun
Jul
Aug
Sep
Oct
Nov
Dec
Month
2006
2005

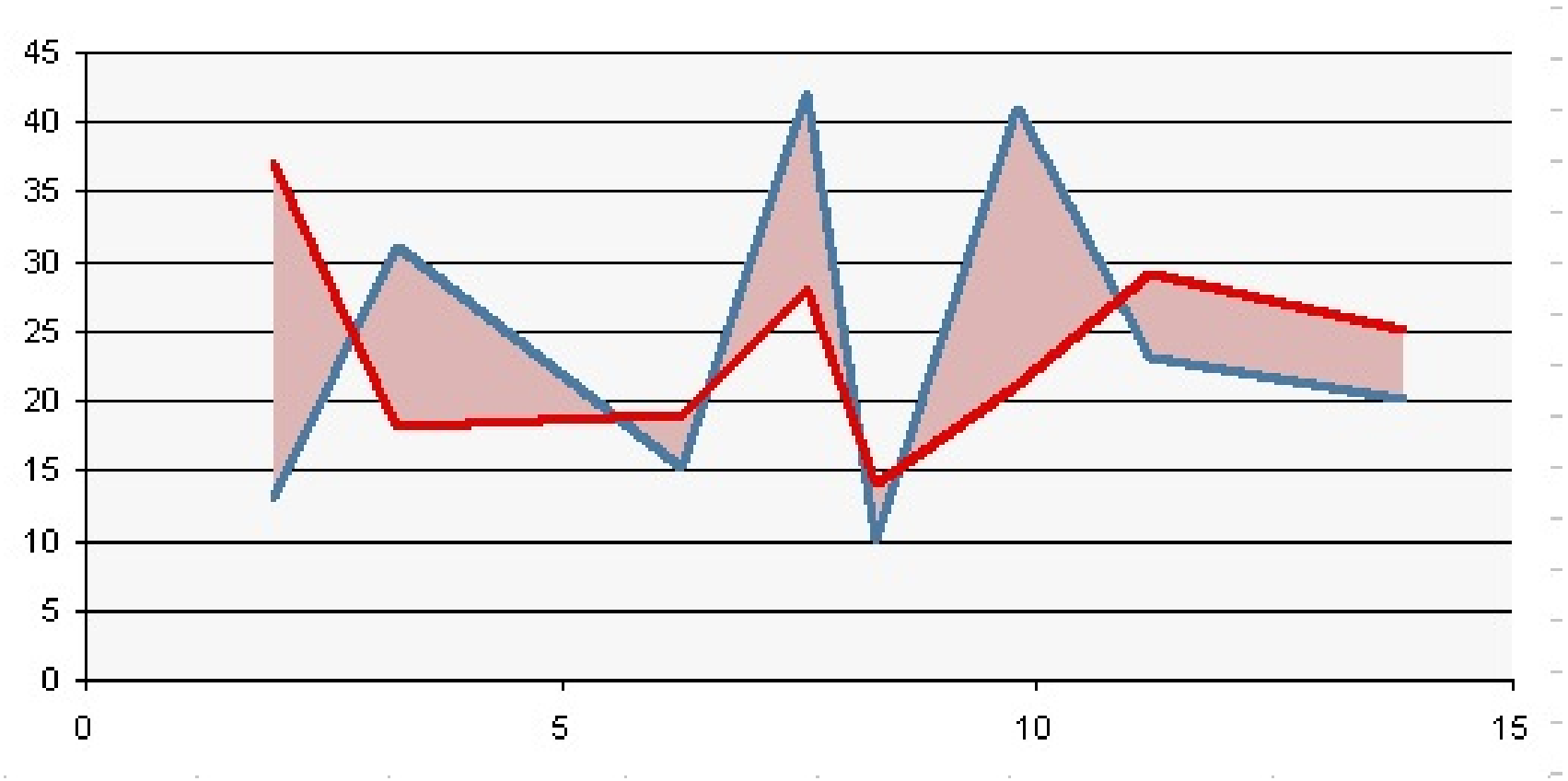
Graphs

Combo



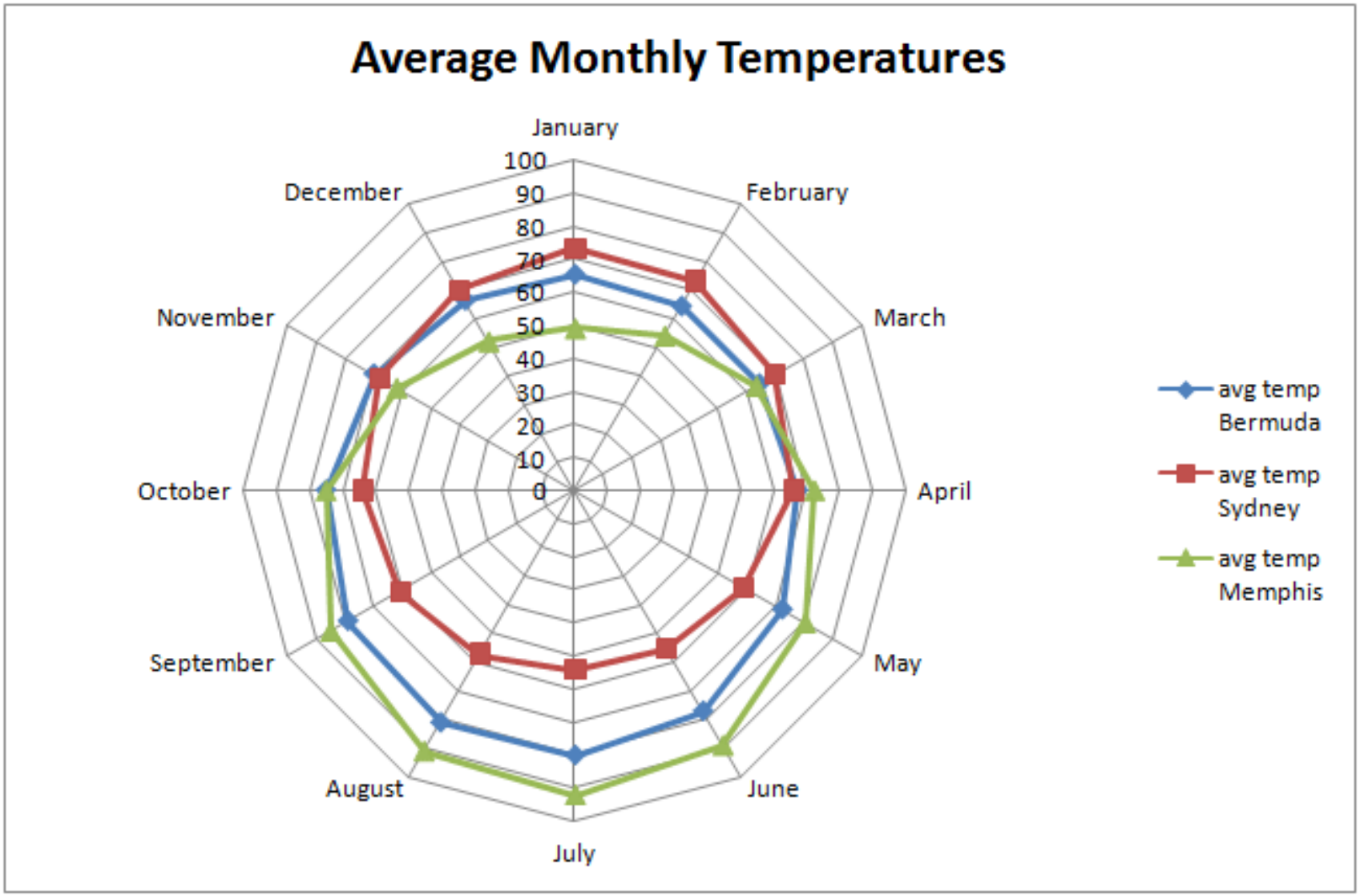
Graphs

Combo



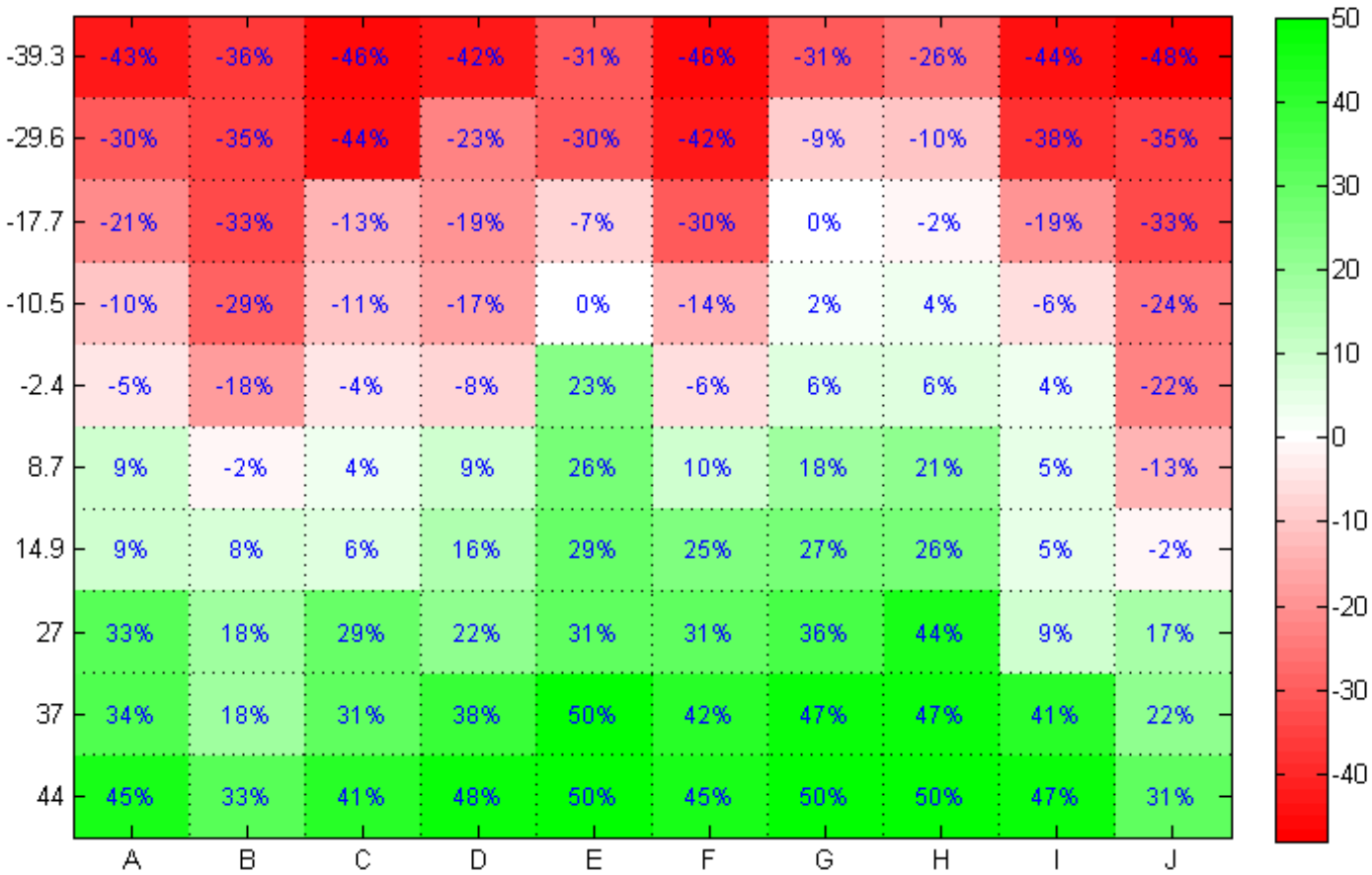
Graphs

Polar chart



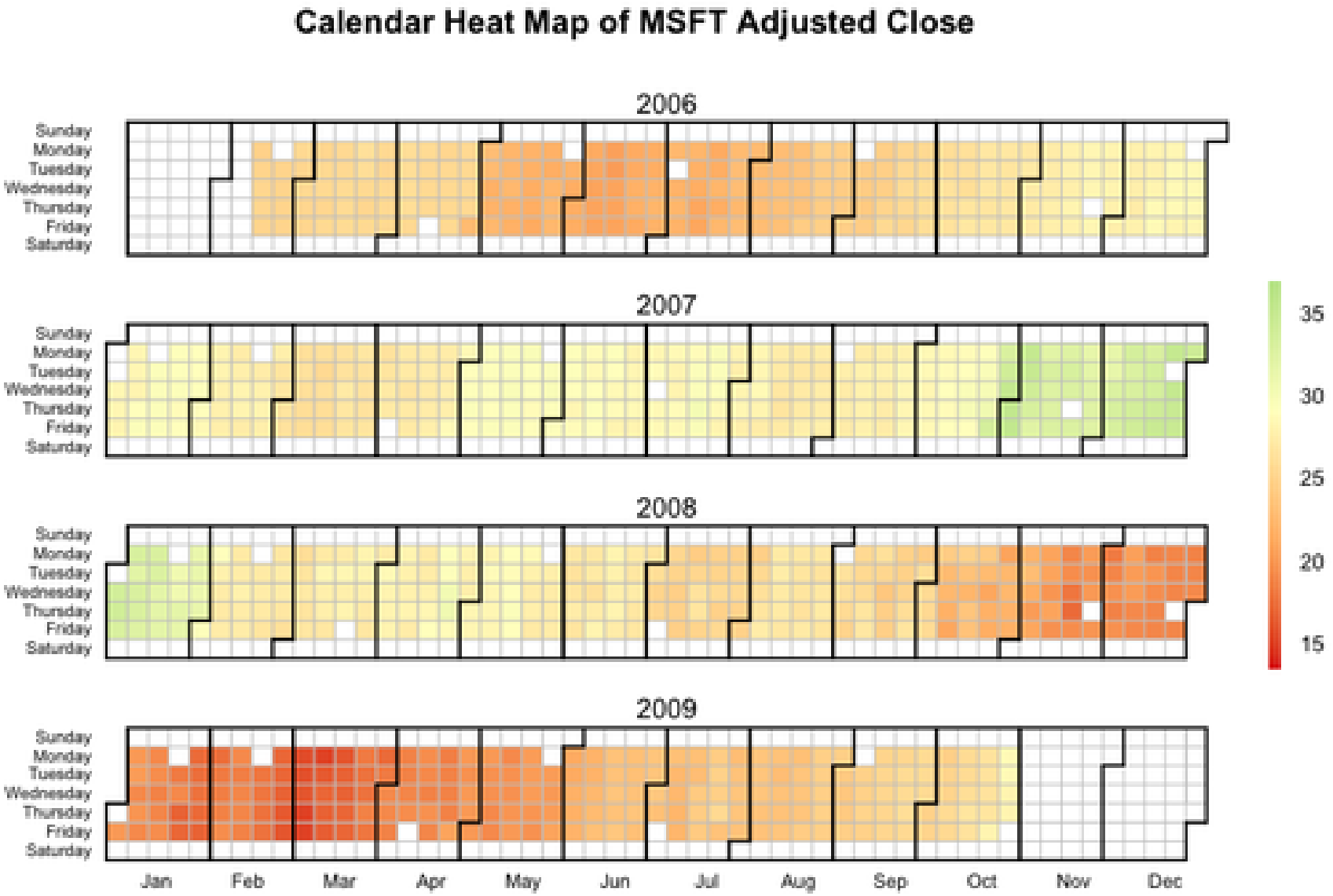
Graphs

Heatmap



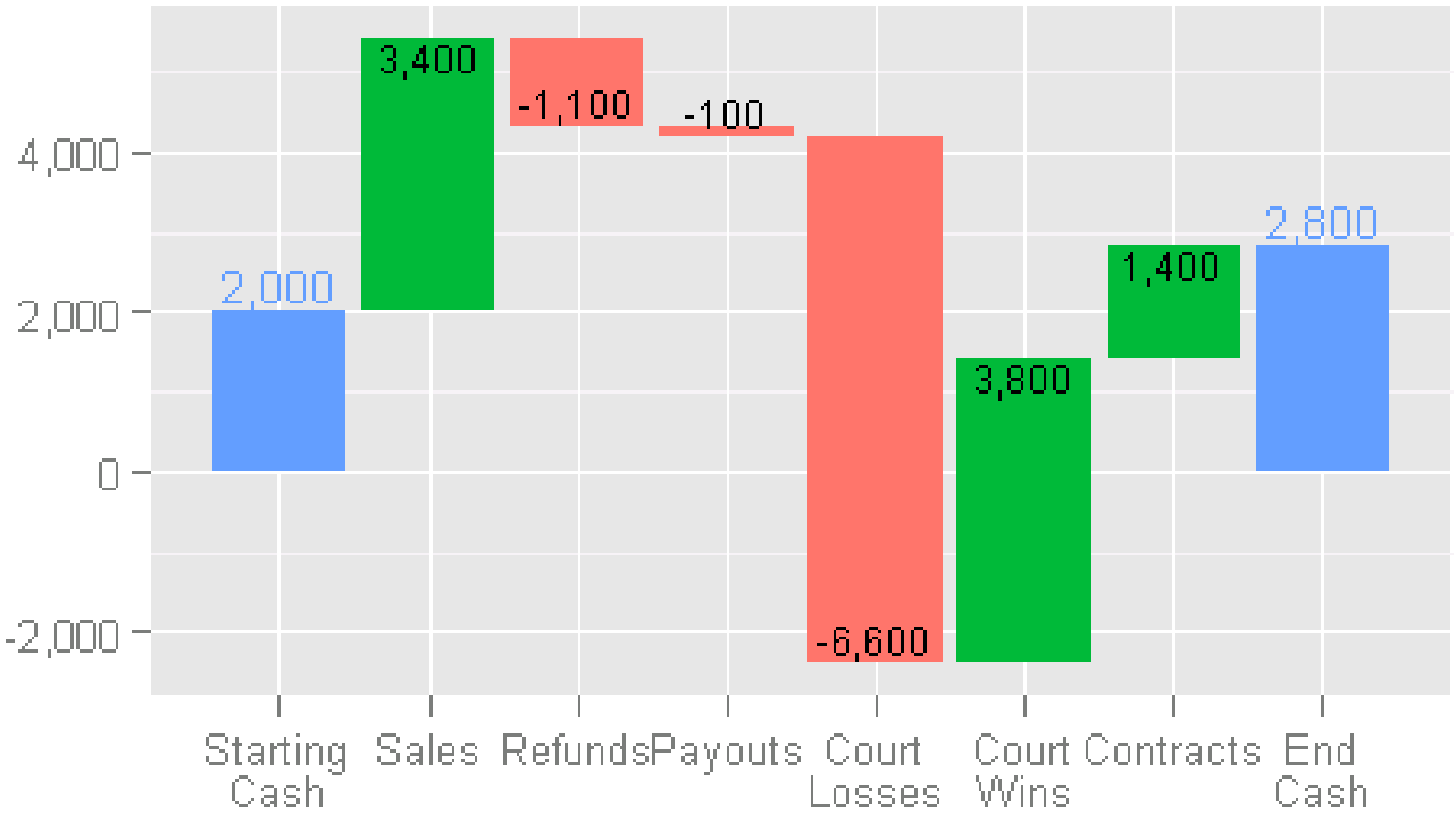
Graphs

Heatmap (calendar)



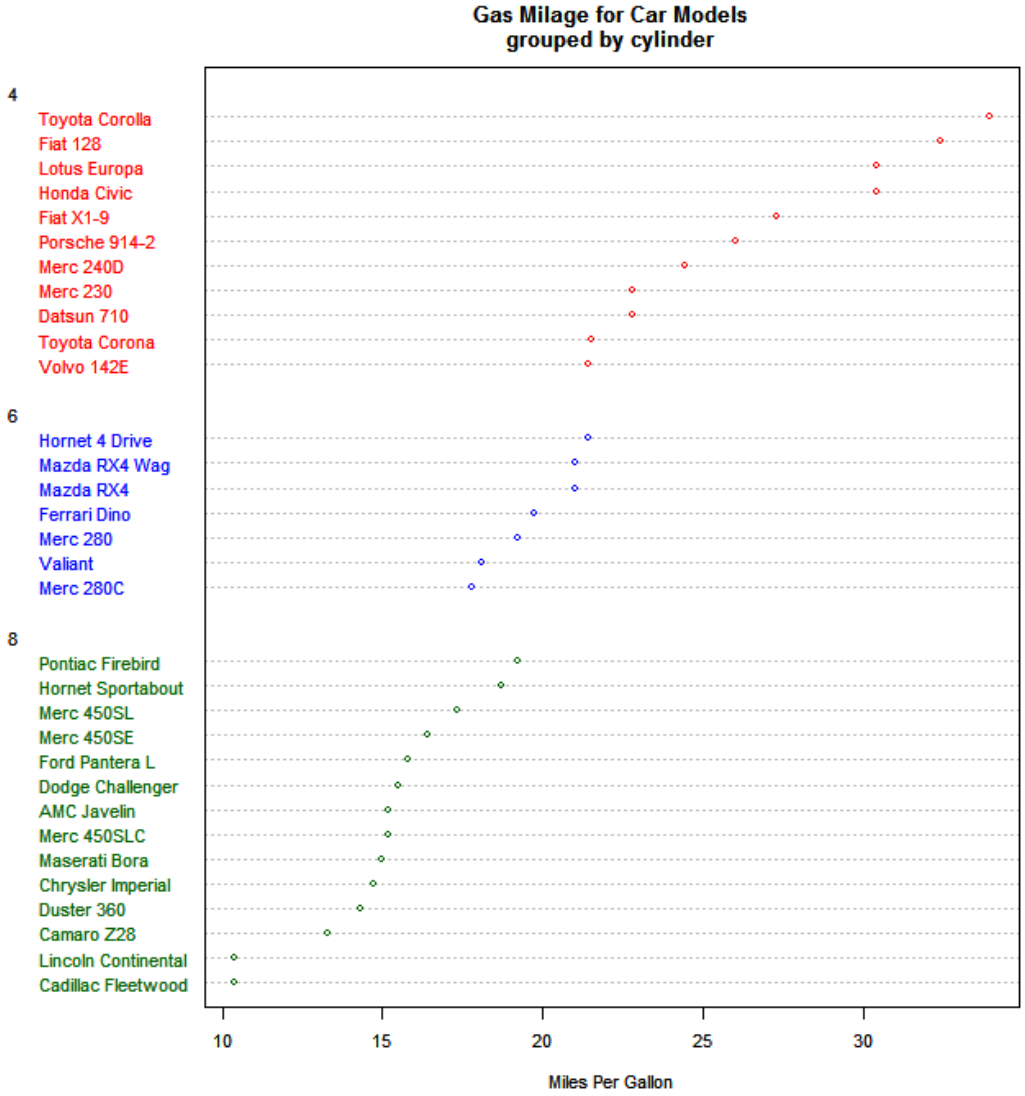
Graphs

Waterfall



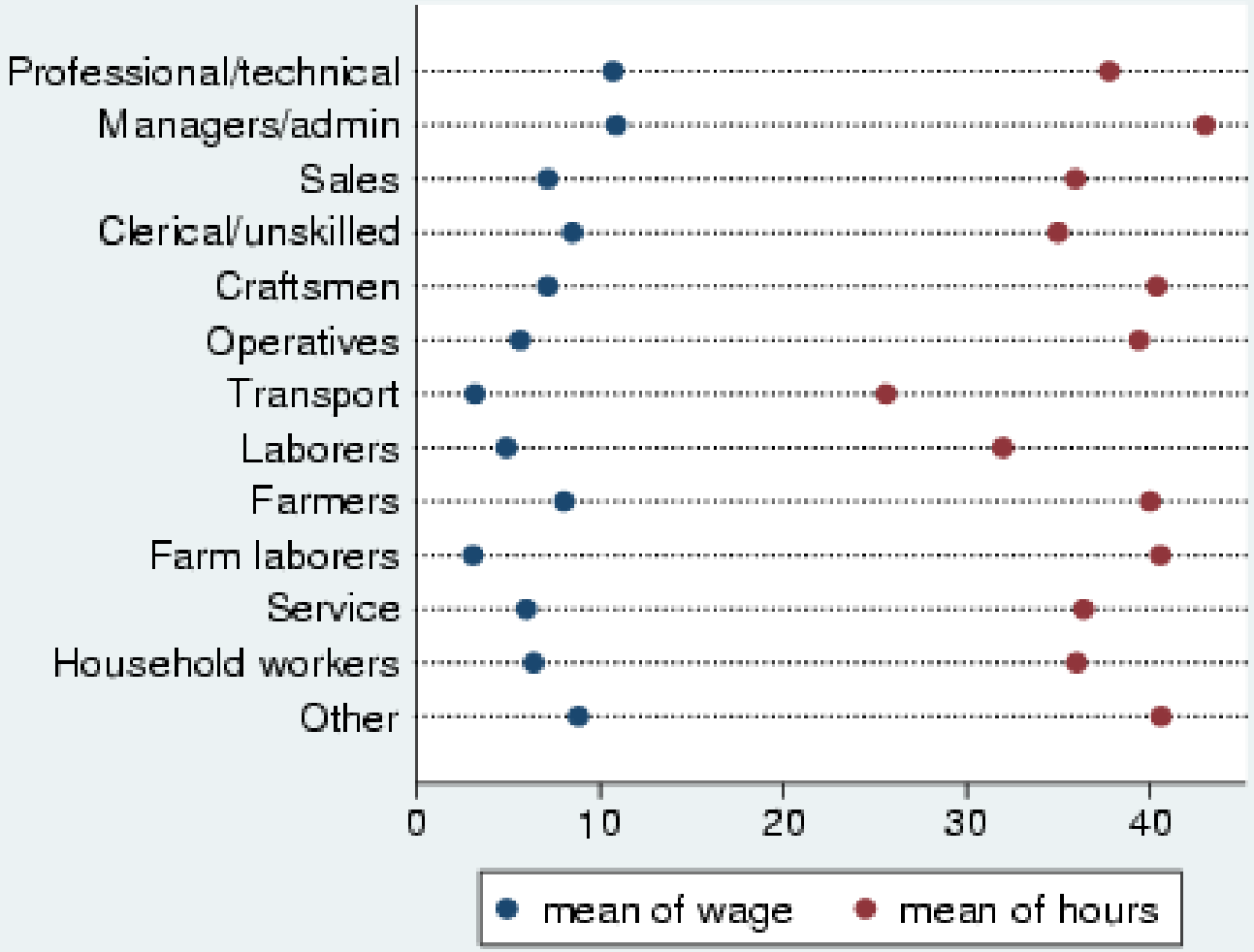
Graphs

Dot plot



Graphs

Dot plot

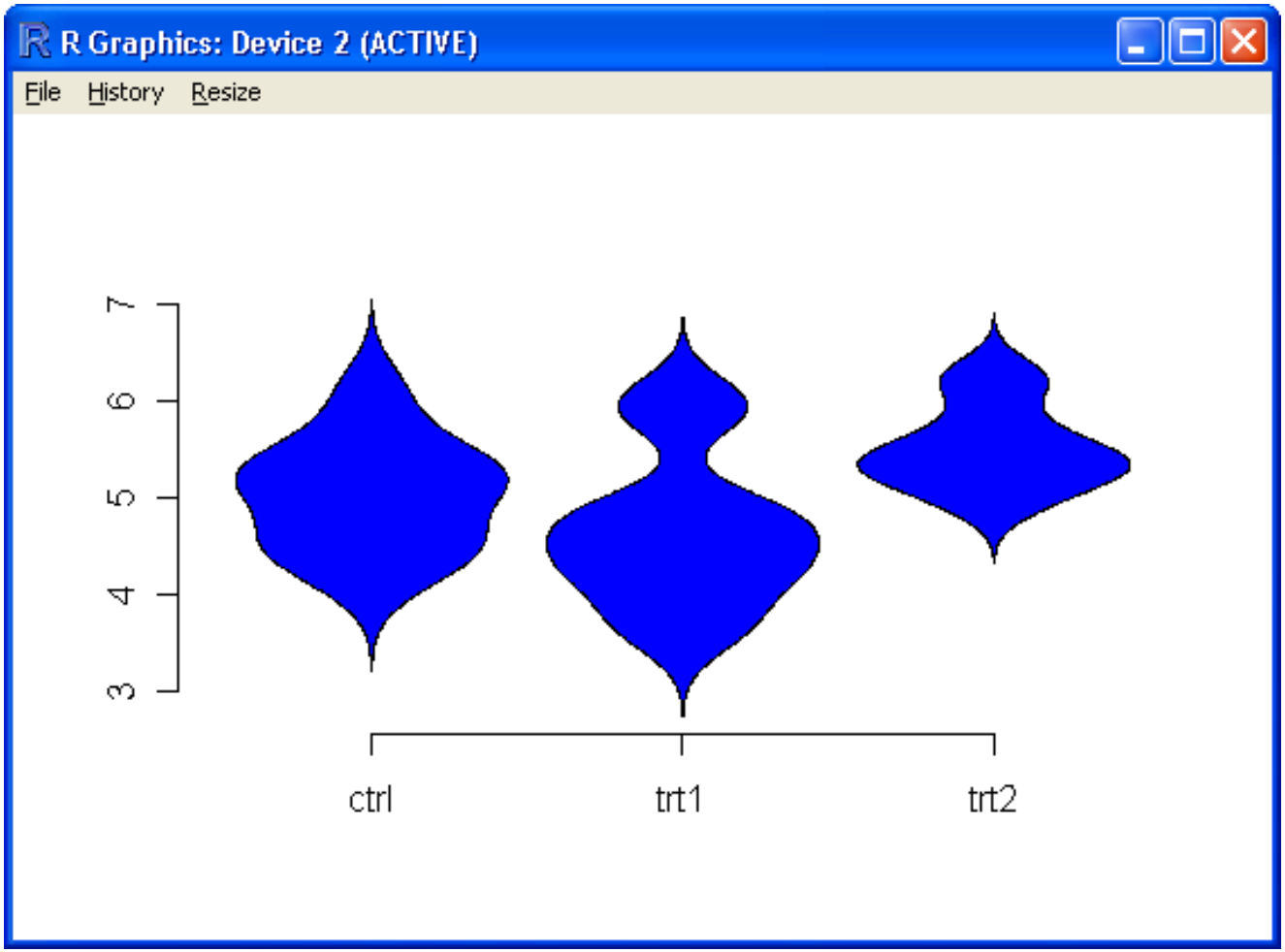


Graphs

Boxplot

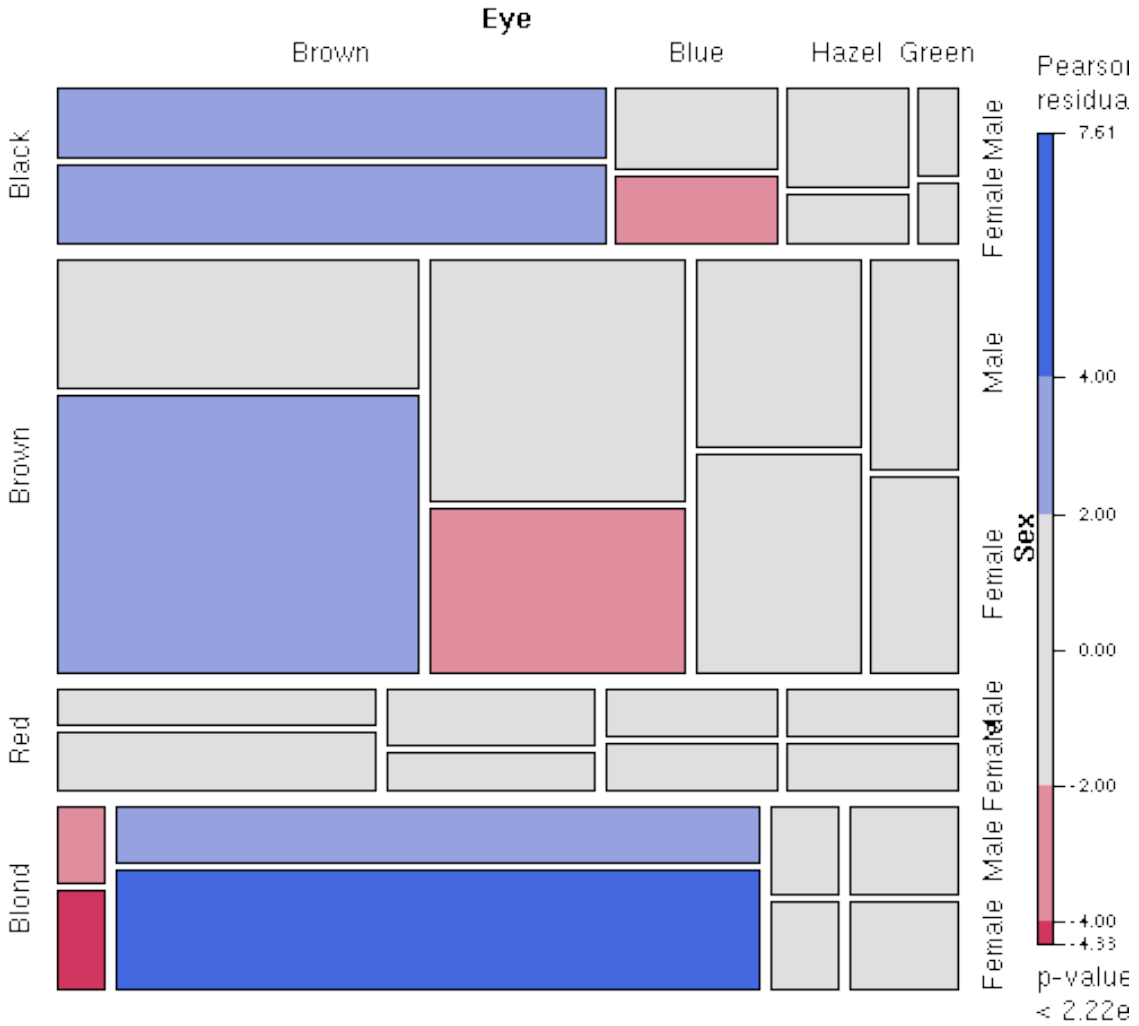
Graphs

Violin plot



Graphs

Mosaic chart

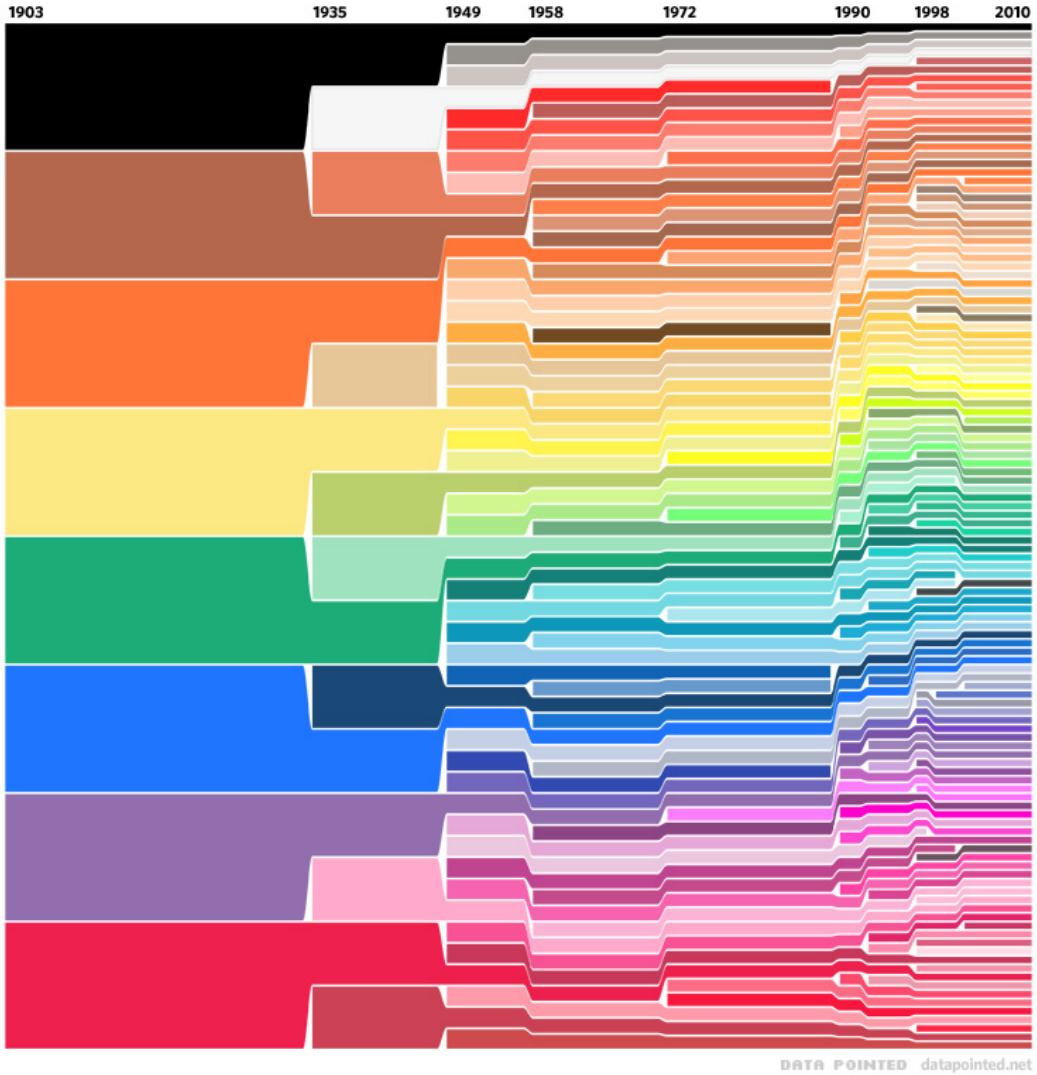


Graphs

Word cloud

Graphs

"Crayola Color Chart, 1903-2010"



Graphs

Some interesting pages about the topic

- http://www.visual-literacy.org/periodic_table/periodic_table.html
- http://www.edwardtufte.com/tufte/
- http://www.perceptualedge.com/
- http://www.visualcomplexity.com/vc/
- http://flowingdata.com/
- http://infosthetics.com/
- http://chartsgraphs.wordpress.com/
- http://www.informationisbeautiful.net/
- http://chartporn.org/

It was a pleasure!

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