

# Quantitative methods

Week #10-11

Gergely Daróczy

Corvinus University of Budapest, Hungary

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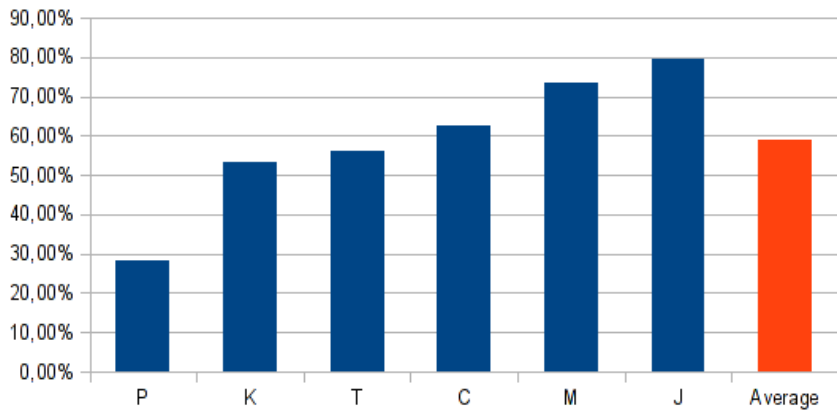


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# Midterm exam

## Results

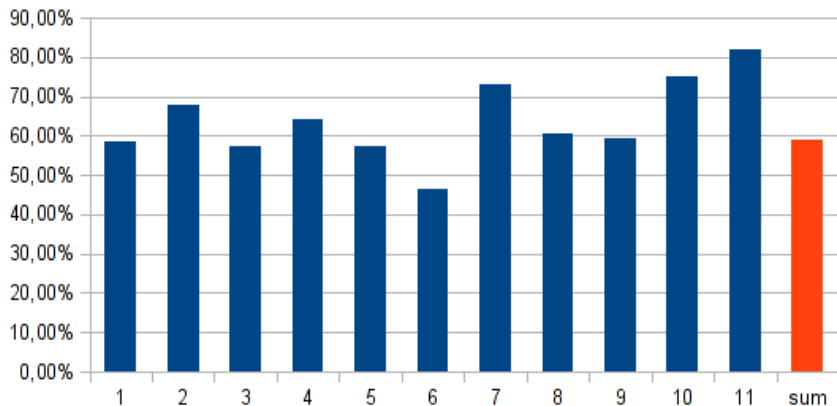
Midterm exam results



# Midterm exam

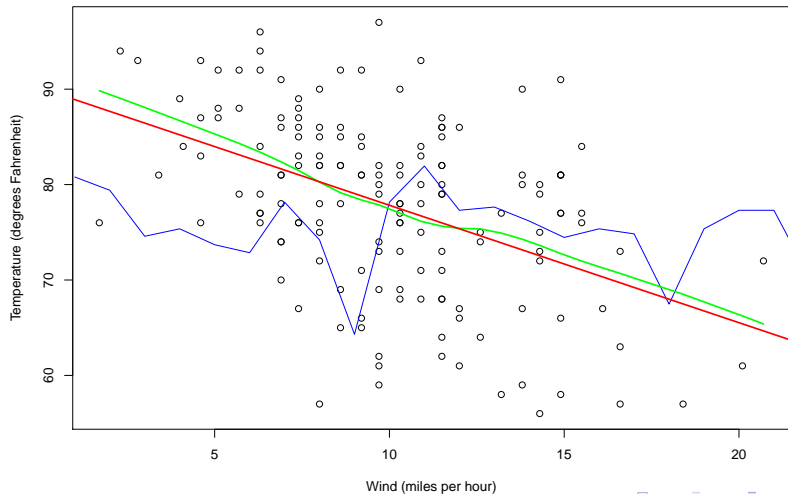
## Results

Midterm exam results



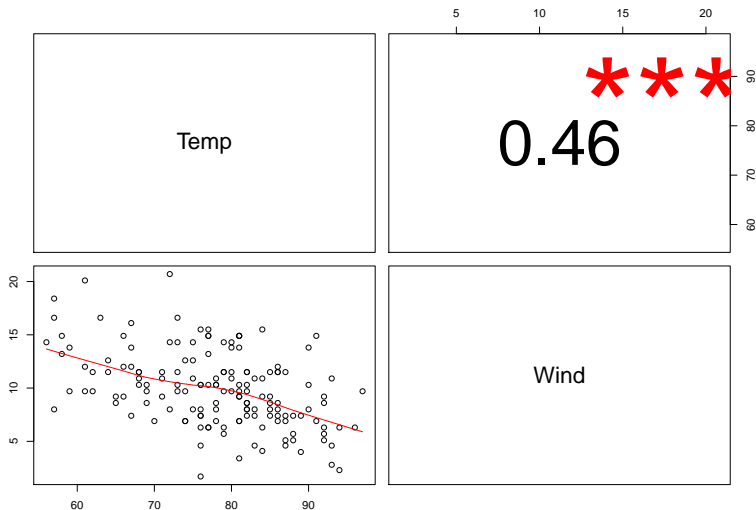
# Correlation

Real association?



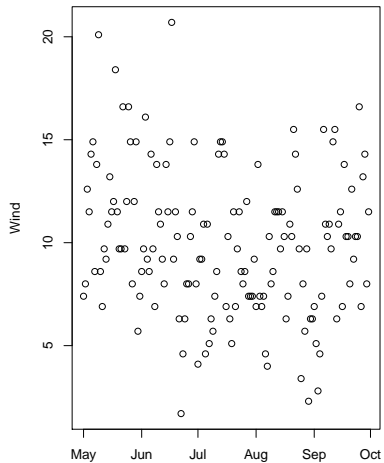
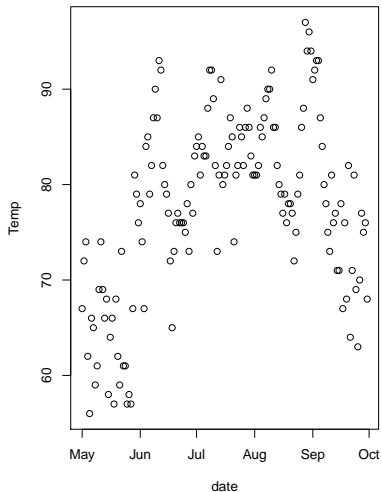
# Correlation

Real association?



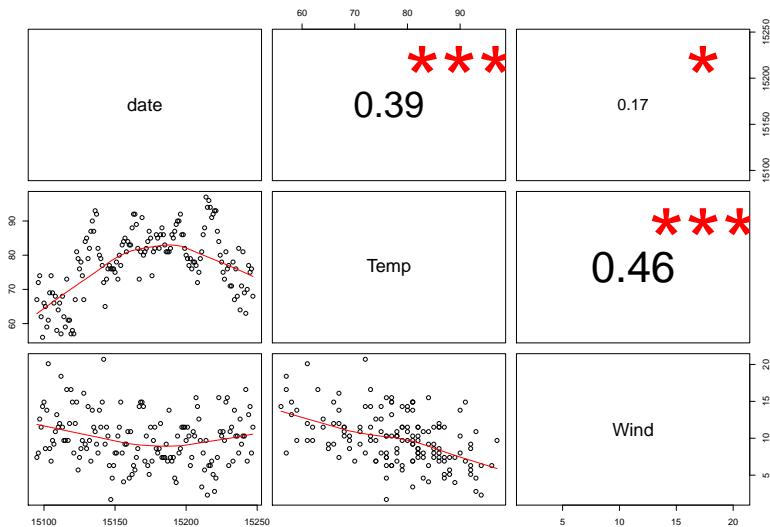
# Correlation

Real association?



# Correlation

Real association?





# Correlation exercise

Required formulas

$$\bar{x} = \frac{\sum_{i=1}^n x_i}{n} \quad s_x = \sqrt{\frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n-1}} \quad \text{COV}(X, Y) = \frac{\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{(n-1)}$$

$$r_{xy} = \frac{\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{(n-1)s_x s_y} = \frac{\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^n (x_i - \bar{x})^2 \sum_{i=1}^n (y_i - \bar{y})^2}}$$

$$\hat{r}_{XY \cdot Z} = \frac{r_{XY} - r_{XZ}r_{YZ}}{\sqrt{(1 - r_{XZ}^2)(1 - r_{YZ}^2)}}$$

$$\hat{\alpha} = \bar{y} - \hat{\beta}\bar{x} \quad \hat{\beta} = \frac{\text{COV}(x, y)}{\text{VAR}(x)} \quad \hat{y} = \beta x + \alpha \quad \hat{y} = \alpha + \beta x$$

**Compute the correlation and build linear models:**

Grade	Monthly scholarship	Spent on books
3	22 000 Ft	4 000 Ft
4	24 000 Ft	3 000 Ft
5	27 000 Ft	2 500 Ft
3.5	24 000 Ft	3 500 Ft
2	23 000 Ft	2 000 Ft

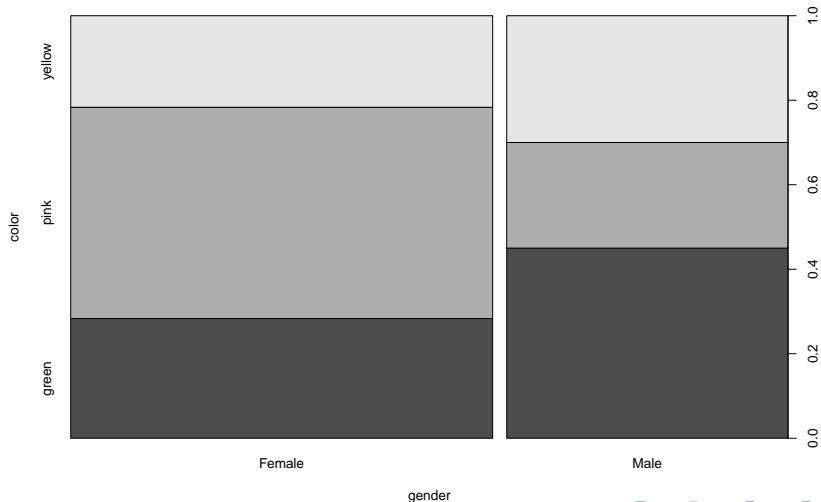
# Crosstables

Discrete (qualitative) variables

ID	gender	color
1	Female	pink
2	Female	pink
3	Female	pink
4	Female	pink
5	Female	pink
6	Female	pink
	...	
95	Male	yellow
96	Male	yellow
97	Male	yellow
98	Male	yellow
99	Male	yellow
100	Male	yellow

# Crosstables

Discrete (qualitative) variables



# Crosstables

Discrete (qualitative) variables

		color		
		green	pink	yellow
gender	Female			
	Male			

# Crosstables

Discrete (qualitative) variables

	green	pink	yellow
Female	17	30	13
Male	18	10	12

# Crosstables

Discrete (qualitative) variables

	green	pink	yellow	
Female	17	30	13	Marginals
Male	18	10	12	
	Marginals			N

# Crosstables

Discrete (qualitative) variables

	green	pink	yellow	$\Sigma$
Female	17	30	13	60
Male	18	10	12	40
$\Sigma$	35	40	25	100



# Crosstables

## Percentages

	green	pink	yellow	$\Sigma$
Female	17	30	13	60
Male	18	10	12	40
$\Sigma$	35	40	25	100

Table : Counted values

	green	pink	yellow	$\Sigma$
Female	17 %	30 %	13 %	60 %
Male	18 %	10 %	12 %	40 %
$\Sigma$	35 %	40 %	25 %	100 %

Table : Total percentages

# Crosstables

## Row percentages

	green	pink	yellow	$\Sigma$
Female	17	30	13	60
Male	18	10	12	40
$\Sigma$	35	40	25	100

Table : Counted values

	green	pink	yellow	$\Sigma$
Female	28.3 %	50 %	21.7 %	100 %
Male	45 %	25 %	30 %	100 %
$\Sigma$	35 %	40 %	25 %	100 %

Table : Row percentages

# Crosstables

## Column percentages

	green	pink	yellow	$\Sigma$
Female	17	30	13	60
Male	18	10	12	40
$\Sigma$	35	40	25	100

Table : Counted values

	green	pink	yellow	$\Sigma$
Female	48.63 %	75 %	52 %	60 %
Male	51.4 %	25 %	48 %	40 %
$\Sigma$	100 %	100 %	100 %	100 %

Table : Column percentages

# Crosstables

## Expected values

	green	pink	yellow	$\Sigma$
Female	17	30	13	60
Male	18	10	12	40
$\Sigma$	35	40	25	100

Table : Counted values

	green	pink	yellow	$\Sigma$
Female	21	24	15	60
Male	14	16	10	40
$\Sigma$	35	40	25	100

Table : Expected values

$$\chi^2 = \sum_{i=1}^n \frac{(O_i - E_i)^2}{E_i}$$

where:

- $\chi^2$ : Pearson's cumulative test statistic,
- $O_i$ : an observed (counted) frequency,
- $E_i$ : an expected (theoretical) frequency,
- $n$ : the number of cells in the table.

$H_0$ : observed and expected values are all the same

Requirements!

# Crosstables

## Computed chi-square

	green	pink	yellow	$\Sigma$
Female	$\frac{(17-21)^2}{21}$	$\frac{(30-24)^2}{24}$	$\frac{(13-15)^2}{15}$	-
Male	$\frac{(18-14)^2}{14}$	$\frac{(10-16)^2}{16}$	$\frac{(12-10)^2}{10}$	-
$\Sigma$	-	-	-	-

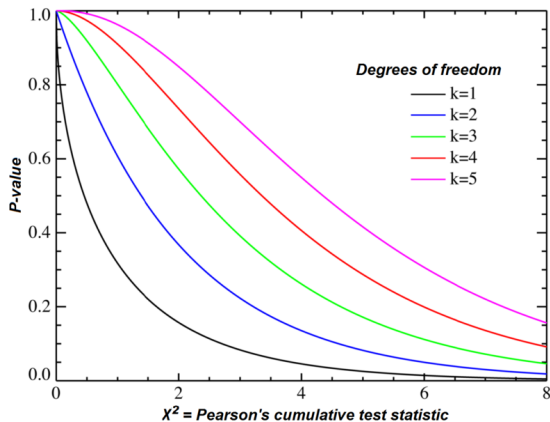
Table : Computed distances between observed and expected values

$$\chi^2 = \sum_{i=1}^n \frac{(O_i - E_i)^2}{E_i} = 6.321429$$

degrees of freedom:  $(2 - 1)(231) = 2$

# Crosstables

## Computed chi-square



$$\chi_c^2 = 2$$

$$\Rightarrow p = 0.04239545$$

# Crosstables

## Exercise

	Read required readings	Did not read required readings
3	15	5
4	20	10
5	45	5

$$E_{i,j} = \frac{M_{i.} \cdot M_{.j}}{N}$$

$$\chi^2 = \sum_{i=1}^n \frac{(O_i - E_i)^2}{E_i}$$

$$df = (3 - 1)(2 - 1) = 2$$

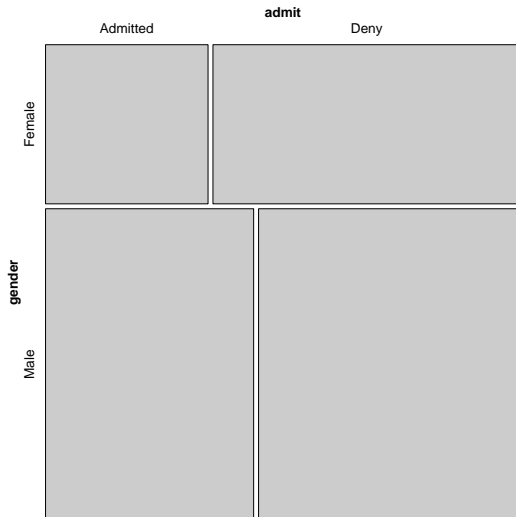
$$\chi_c^2 = 2$$

$$\phi = \frac{\chi}{N} \quad V_c = \sqrt{\frac{\chi^2}{N \cdot (k - 1)}}$$



# Simpson's paradox

Berkeley sex bias case



# Simpson's paradox

Berkeley sex bias case

	Admitted	Deny	$\Sigma$
Female	1494	2827	4321
Male	3738	4704	8442
$\Sigma$	5232	7531	12763

Table : Observed values

# Simpson's paradox

Berkeley sex bias case

	Admitted	Deny	$\Sigma$
Female	1494	2827	4321
Male	3738	4704	8442
$\Sigma$	5232	7531	12763

Table : Observed values

	Admitted	Deny	$\Sigma$
Female	34.6 %	65.4 %	100 %
Male	44.3 %	55.7 %	100 %
$\Sigma$	41 %	59 %	100 %

Table : Row percentages

# Simpson's paradox

## Berkeley sex bias case

	Admitted	Deny	$\Sigma$
Female	1494	2827	4321
Male	3738	4704	8442
$\Sigma$	5232	7531	12763

Table : Observed values

	Admitted	Deny	$\Sigma$
Female	34.6 %	65.4 %	100 %
Male	44.3 %	55.7 %	100 %
$\Sigma$	41 %	59 %	100 %

Table : Row percentages

$$\chi^2 = 110.8489; d.f. = 1; p = 6.385628e - 26$$

# Simpson's paradox

## Berkeley sex bias case

	Applicants	Admitted
Men	8442	<b>44%</b>
Women	4321	35%

Departement	Men		Women	
	Applicants	Admitted	Applicants	Admitted
A	825	62%	108	<b>82%</b>
B	560	63%	25	<b>68%</b>
C	325	<b>37%</b>	593	34%
D	417	33%	375	<b>35%</b>
E	191	<b>28%</b>	393	24%
F	272	6%	341	<b>7%</b>

# Simpson's paradox

Batting averages in professional baseball

	1995		1996		Combined	
	Runs/Outs	%	Runs/Outs	%	Runs/Outs	%
Derek Jeter	12/48	25 %	183/582	31.4 %	195/630	<b>31 %</b>
David Justice	104/411	<b>25.3 %</b>	45/140	<b>32.1 %</b>	149/551	27 %

Who is the better player?

It was a pleasure!

Gergely Daróczy

*daroczi.gergely@btk.ppke.hu*