Quantitative methods

Lesson 10

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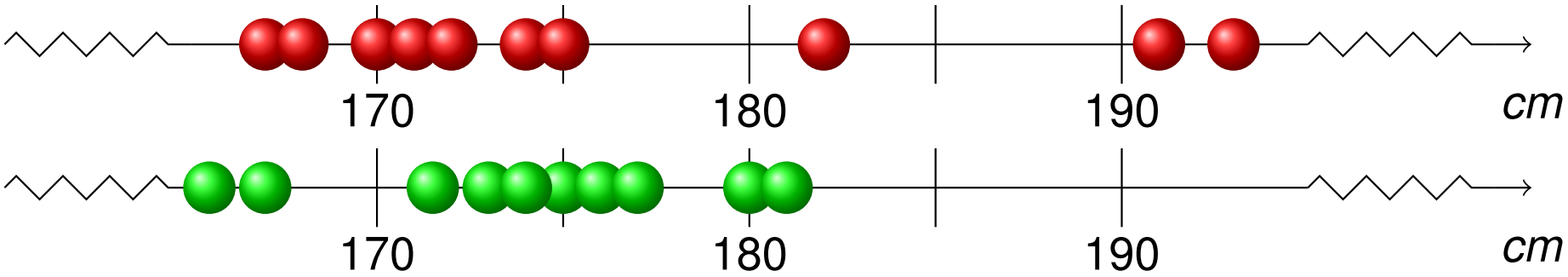
Outline

Repeating

Averages

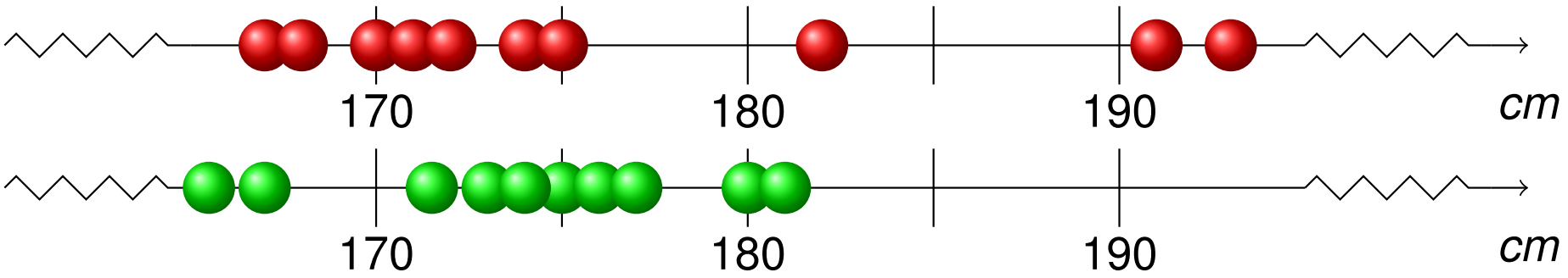
We have measured 10-10 students in two classrooms.

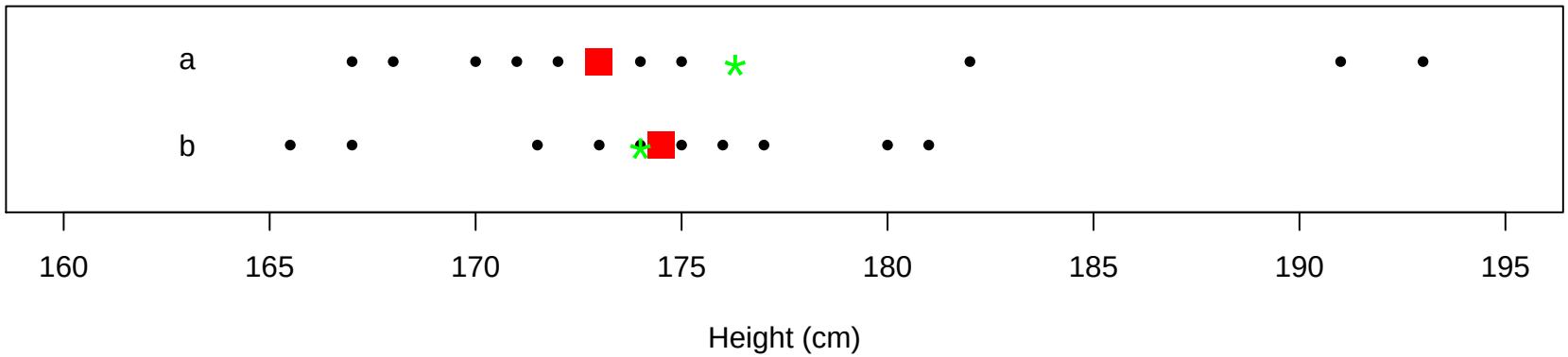


Which class has higher students based on this small sample? Think about averages as good estimates of populatioin parameters!

Repeating

Averages





Research in an elementary school

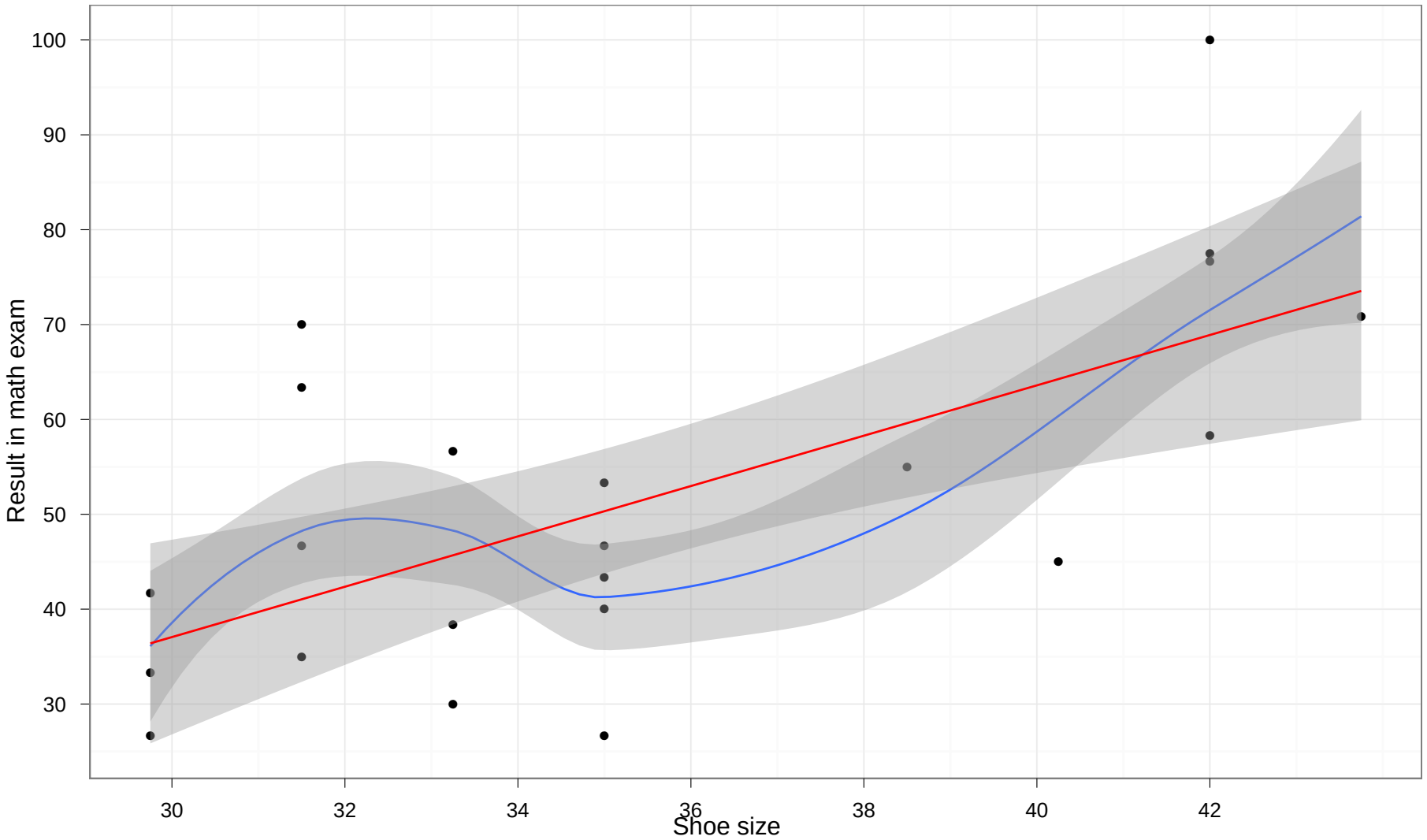
Big shoes and smart kids (example)

We made a small research on the age and shoe size of some students in an elementary shool, where we also conducted a math exam. See detailed results below:

	Shoe size	Math result	Age
1	29.75	26.67	3
2	29.75	33.33	7
3	29.75	41.67	5
4	31.50	35.00	8
5	31.50	46.67	10
6	31.50	63.33	11
7	31.50	70.00	12
8	33.25	30.00	7.
9	33.25	38.33	7
10	33.25	56.67	12
11	35.00	26.67	6
12	35.00	40.00	8
13	35.00	43.33	6
14	35.00	46.67	10
15	35.00	53.33	11
16	38.50	55.00	9
17	40.25	45.00	9
18	42.00	58.33	9
19	42.00	76.67	16
20	42.00	77.50	18
21	42.00	100.00	19
22	43.75	70.83	14

Research in an elementary school

Big shoes and smart kids (example)



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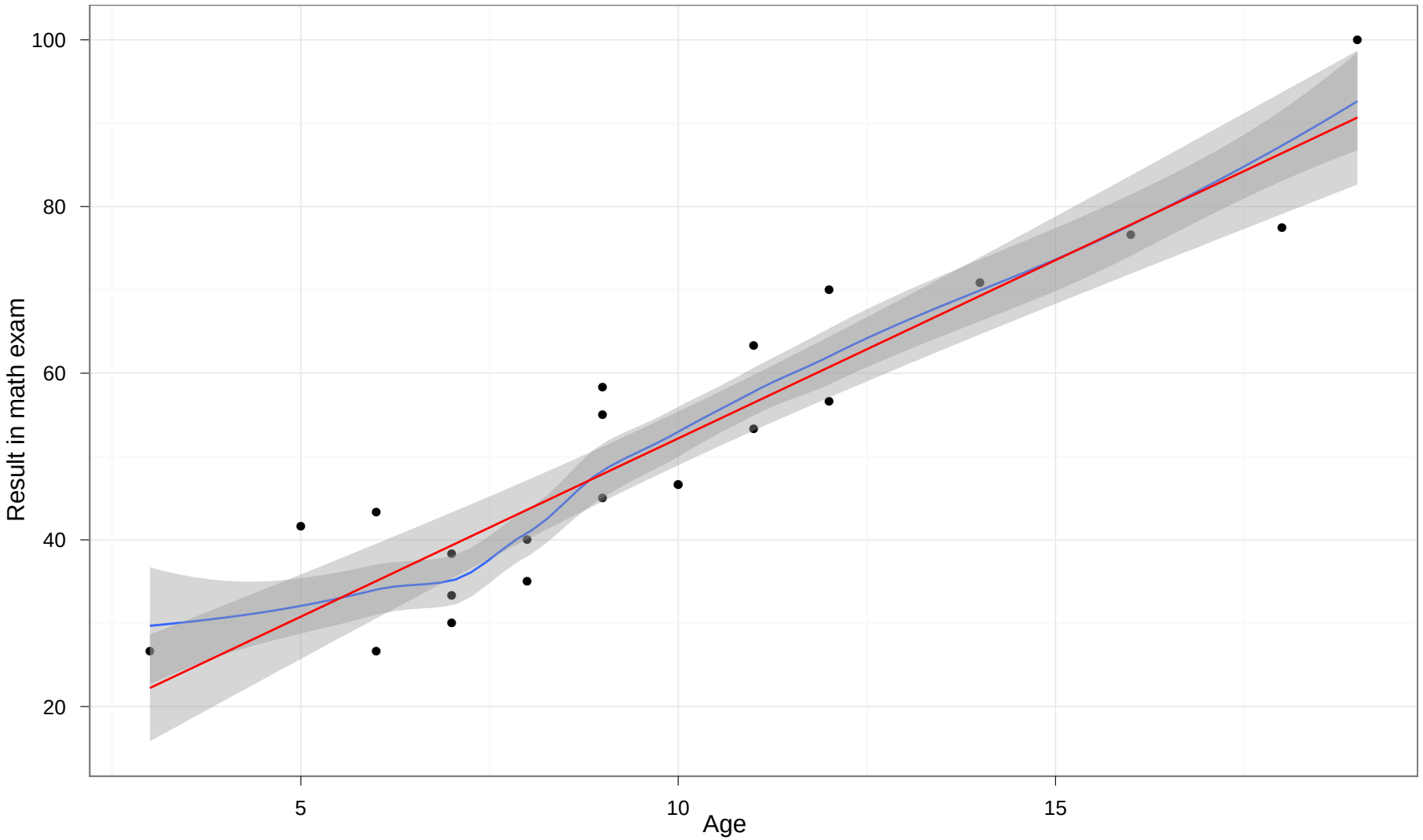

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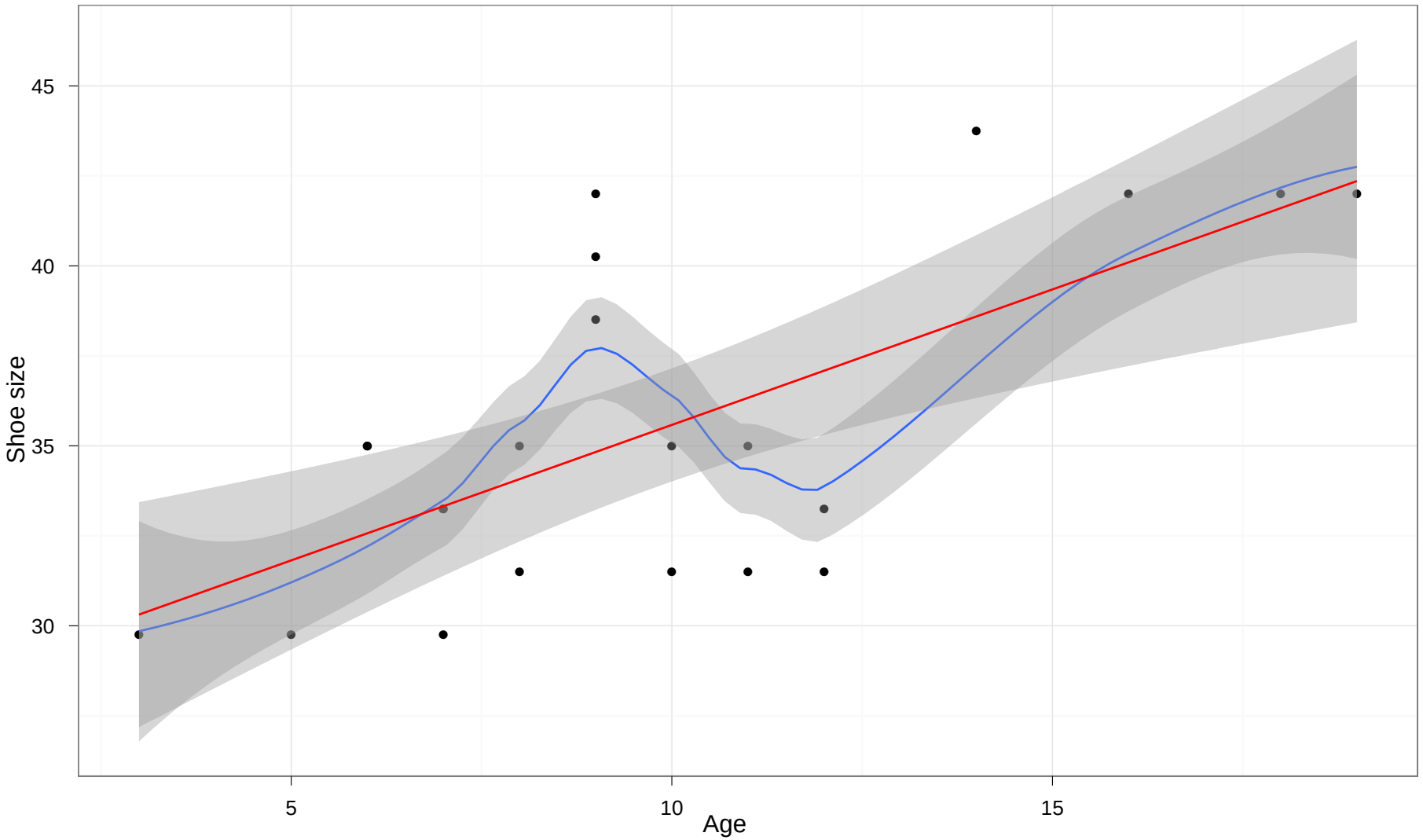

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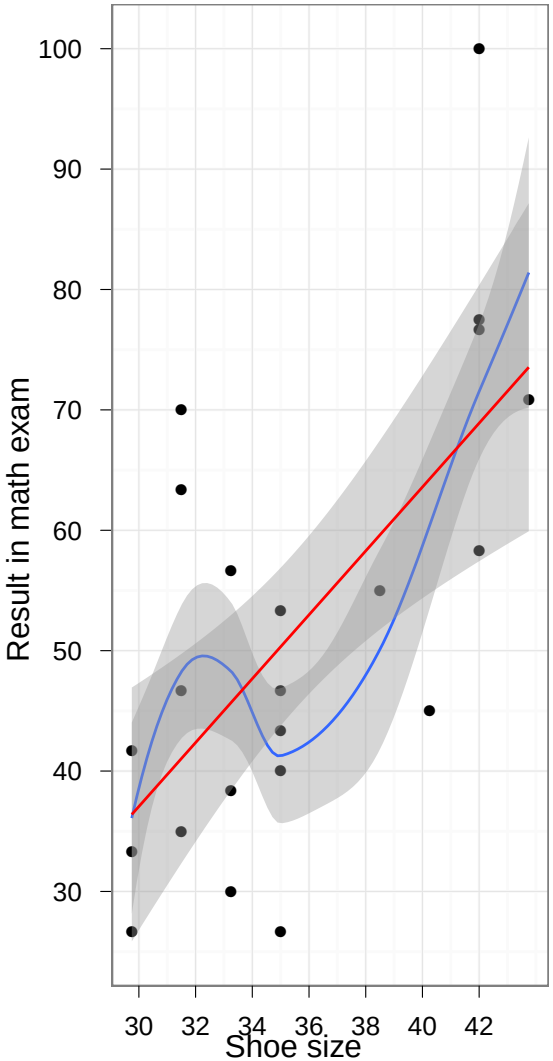
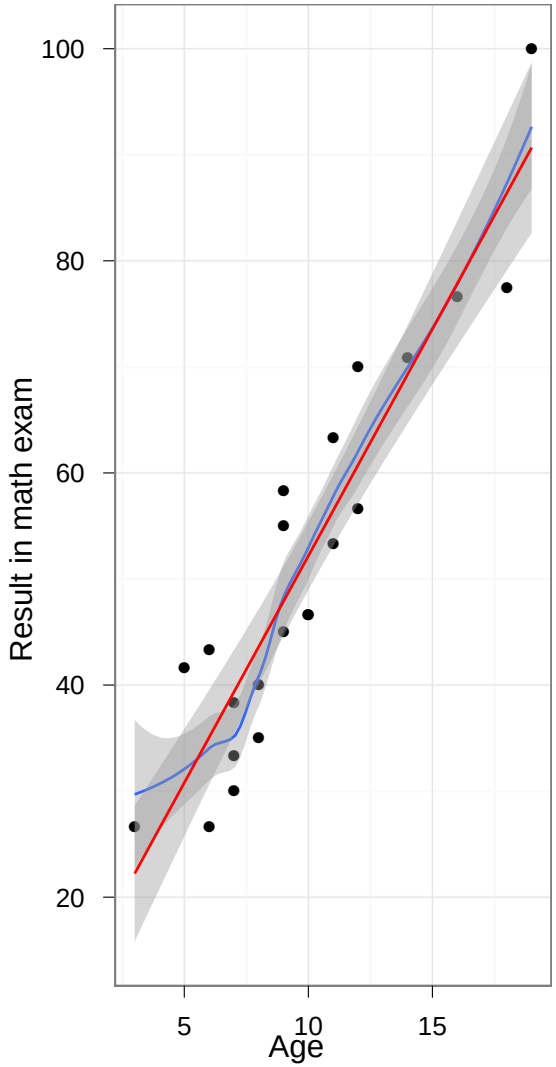
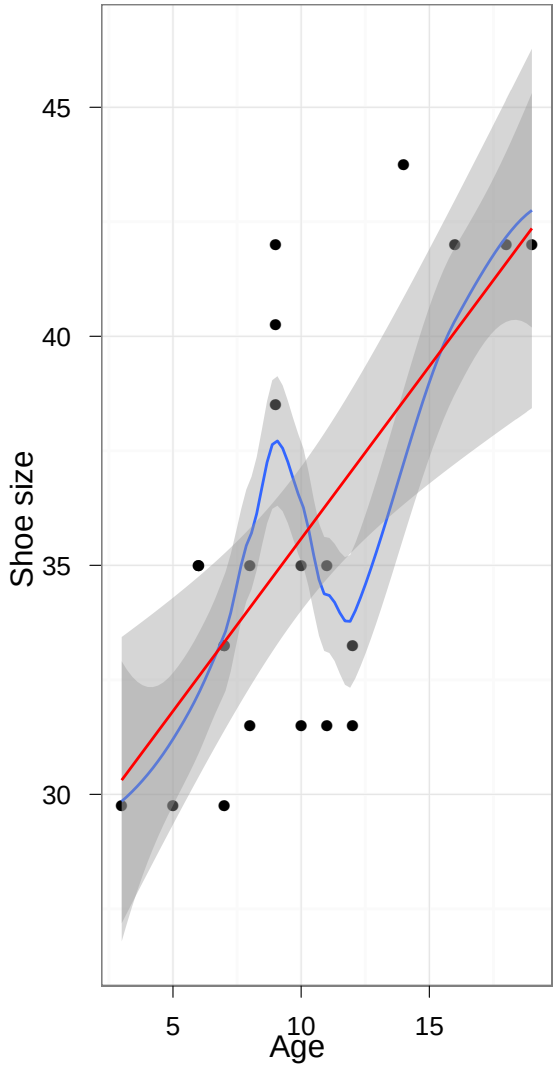
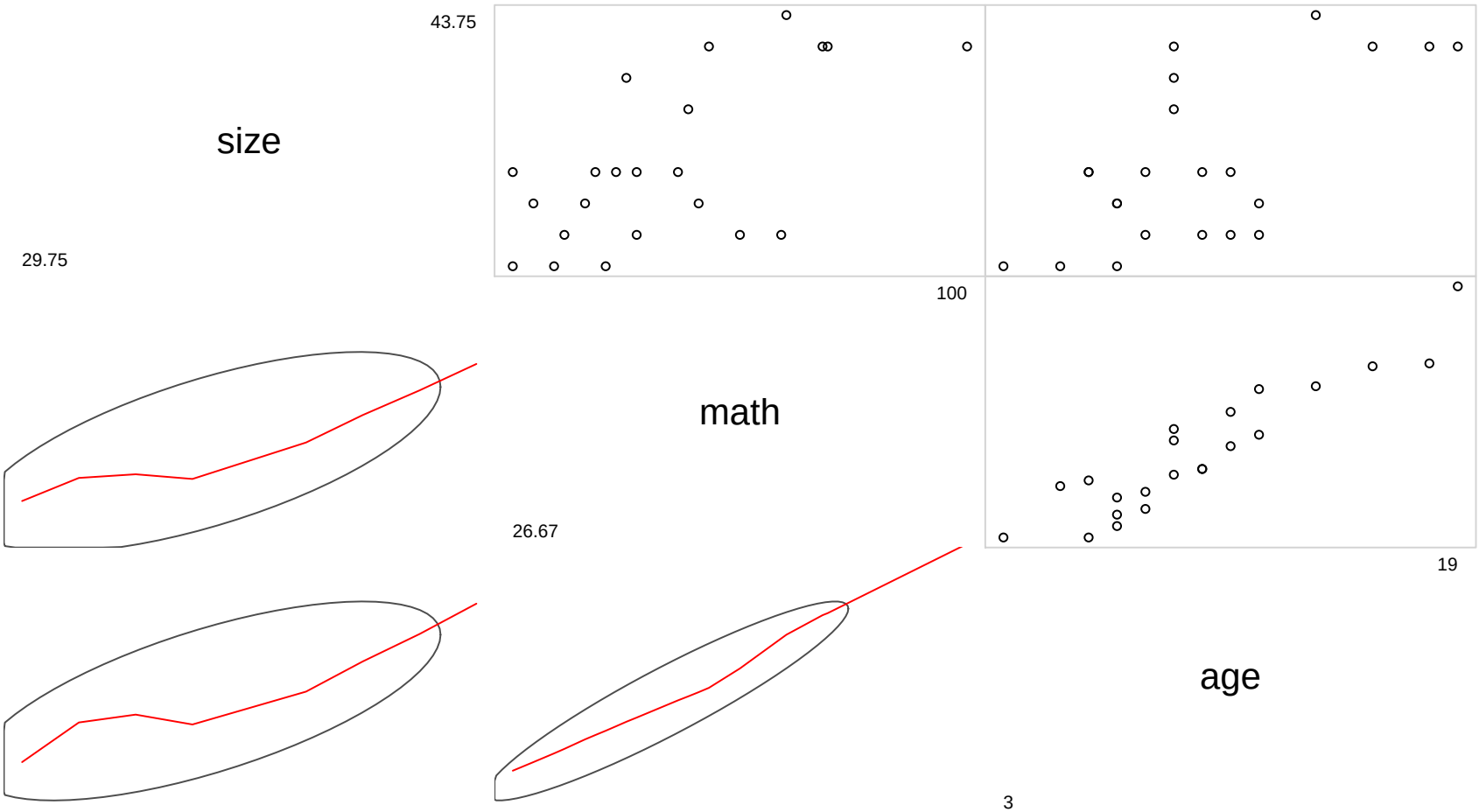






Research in an elementary school

Big shoes and smart kids (example)



Research in an elementary school

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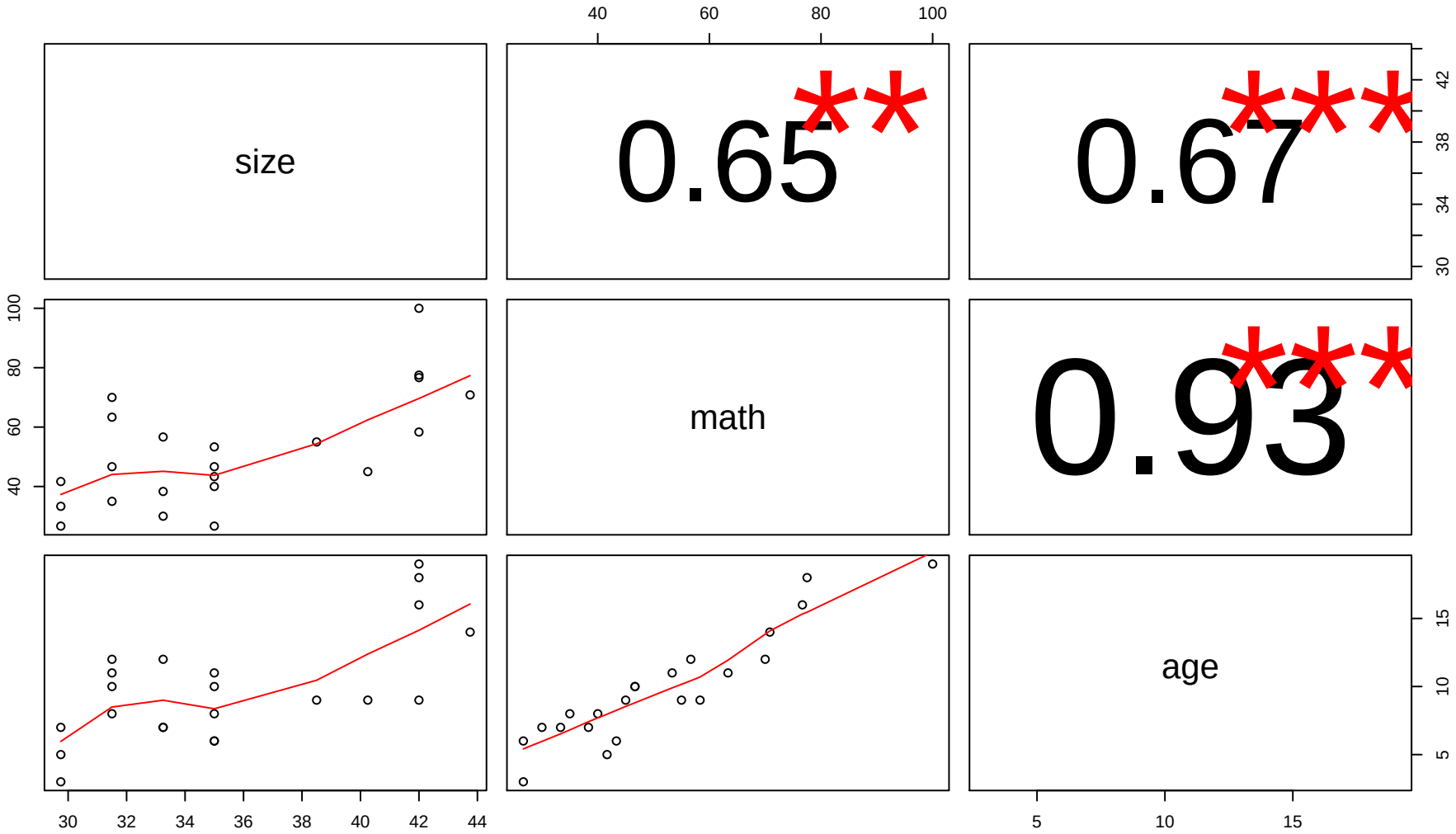


Partial correlation:

$$r_{math,size \cdot age} = 0.11$$

$$r_{math,age \cdot size} = 0.87$$

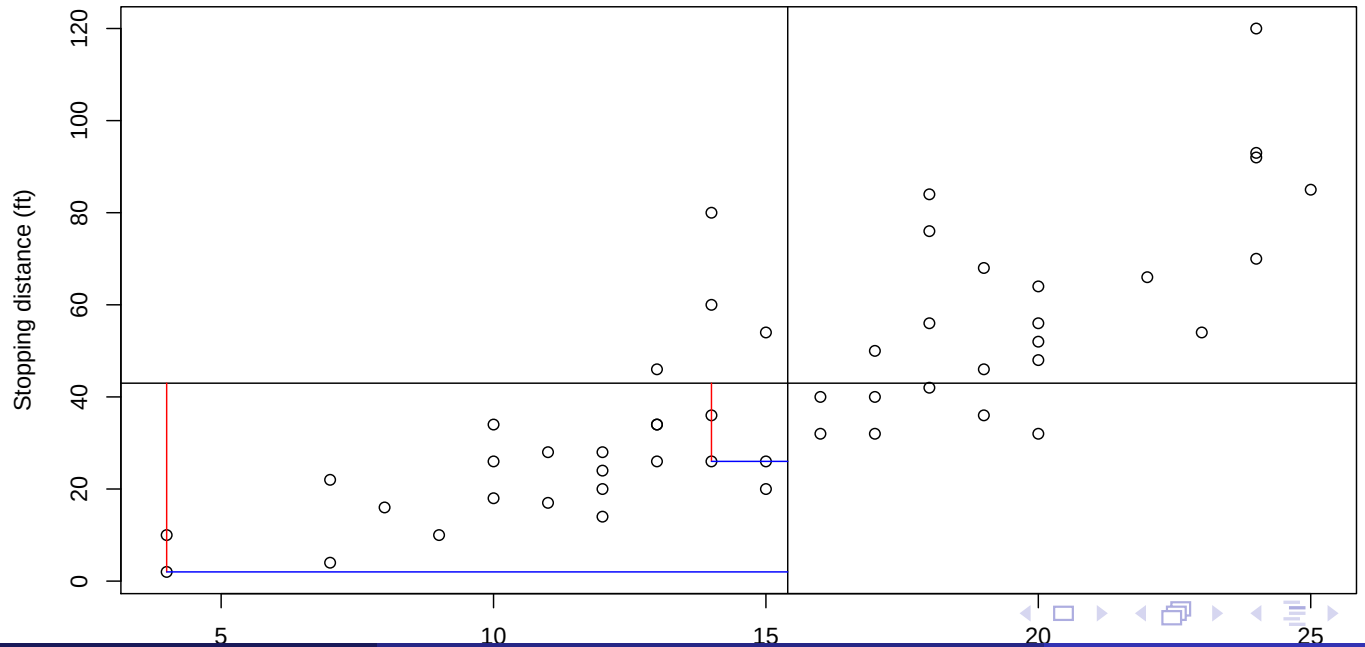
$$r_{size,age \cdot math} = 0.22$$

For x and y variables the joint variablity could be computed by :

$$COV(xy) = \sum_{i=1}^{n} \frac{(x_i - \overline{x})(y_i - \overline{y})}{n-1}$$

$$remember : \sigma = \sqrt{\sum_{i=1}^{n} \frac{(x_i - \overline{x})^2}{n}}$$

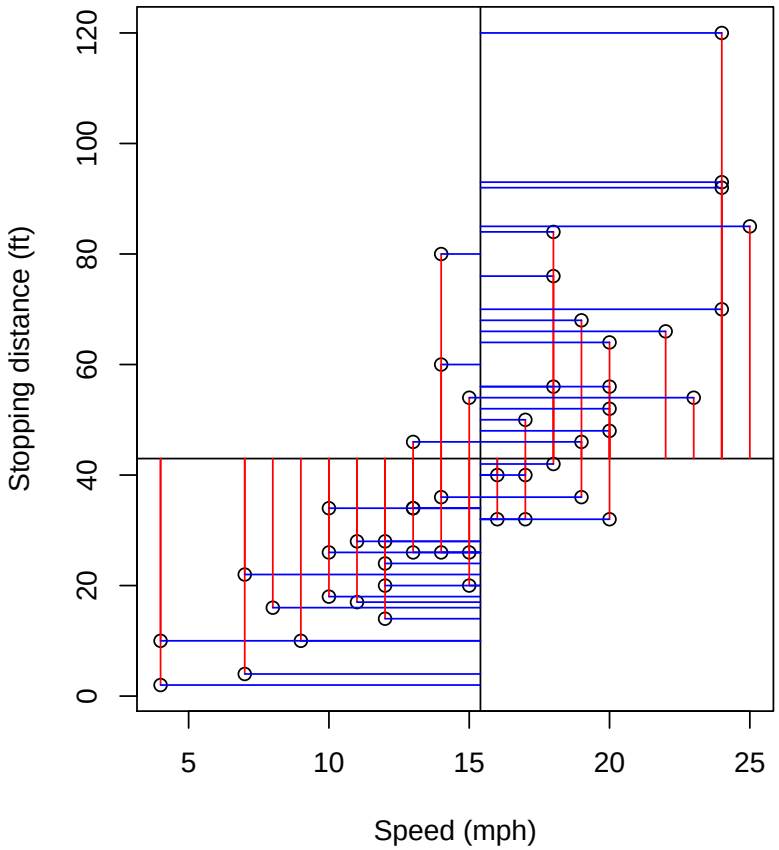
Ezekiel, M. (1930) Methods of Correlation Analysis. Wiley.



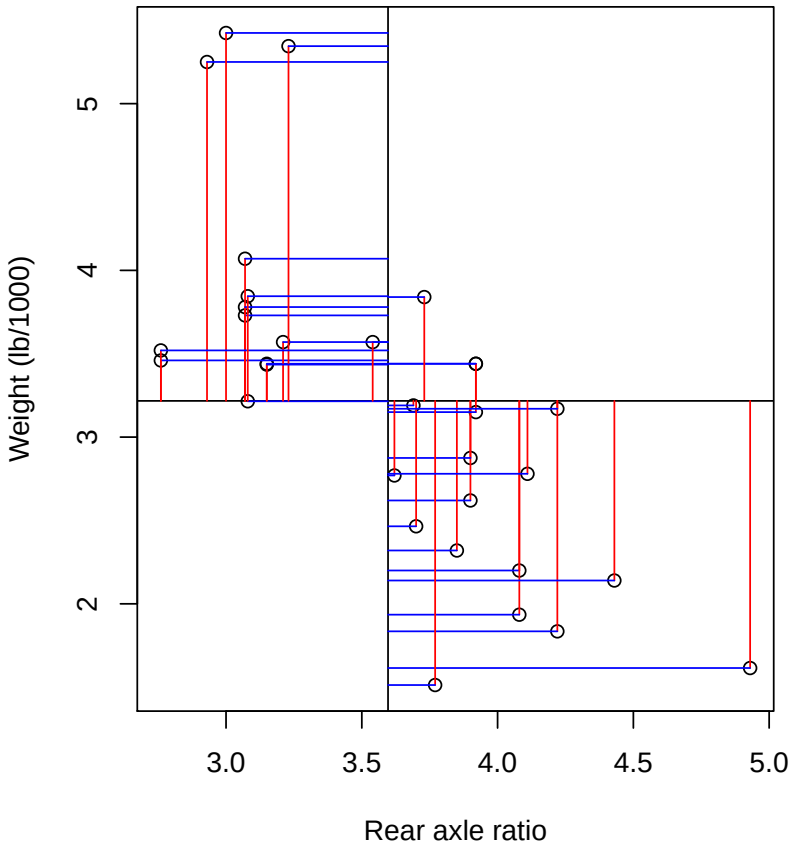
Theoretical background

Covariation

Ezekiel, M. (1930):
Methods of Correlation Analysis



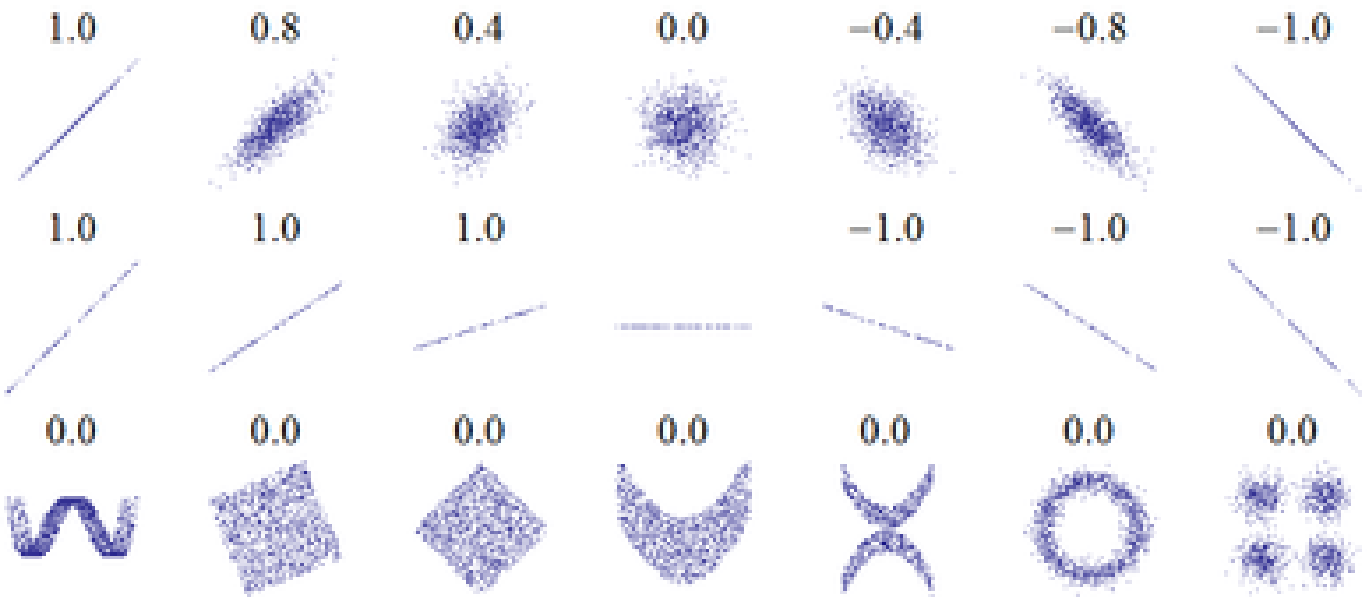
Henderson & Velleman (1981):
Building multiple regression models interactively



Theoretical background

Correlation

$$r_{xy} = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{(n-1)s_x s_y} = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^{n}(x_i - \bar{x})^2 \sum_{i=1}^{n}(y_i - \bar{y})^2}}$$



$$\hat{r}_{XY \cdot \mathbf{Z}} = \frac{N\sum_{i=1}^{N} r_{X,i} r_{Y,i} - \sum_{i=1}^{N} r_{X,i} \sum_{i=1}^{N} r_{Y,i}}{\sqrt{N\sum_{i=1}^{N} r_{X,i}^2 - \left(\sum_{i=1}^{N} r_{X,i}\right)^2} \sqrt{N\sum_{i=1}^{N} r_{Y,i}^2 - \left(\sum_{i=1}^{N} r_{Y,i}\right)^2}}$$

so for three variables:

$$\hat{r}_{XY \cdot \mathbf{Z}} = \frac{r_{XY} - r_{X\mathbf{Z}} r_{Y\mathbf{Z}}}{\sqrt{(1 - r_{X\mathbf{Z}}^2)(1 - r_{Y\mathbf{Z}}^2)}}$$

Exercises

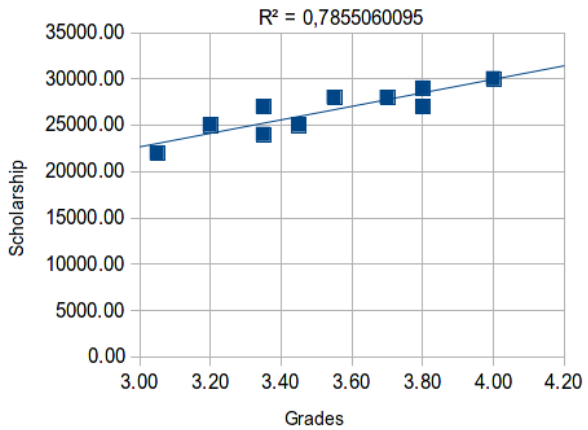
1. What is correlation and partial correlation?
2. Building upon your findings, compute the possible pairs of correlation coefficients on the below dataset!
3. Also look for partial correlation and comment on your results!

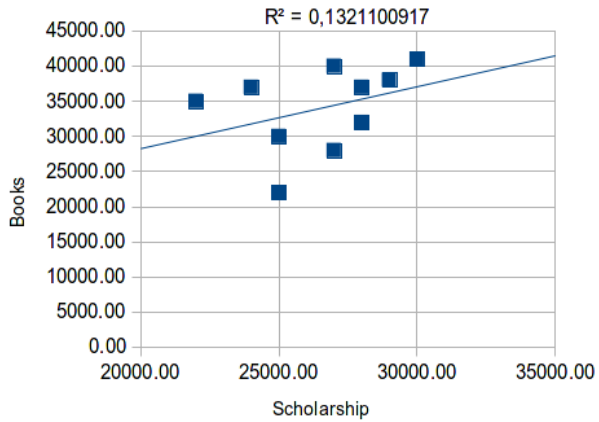
Grade (mean)	Scholarship (in HUF)	Money spent on books (in HUF)
3.05	22000	3500
3.2	25000	3000
3.35	27000	2800
3.35	24000	3700
3.45	25000	2200
3.55	28000	3200
3.7	28000	3700
45	30000	4100
3.8	27000	4000
3.8	29000	3800

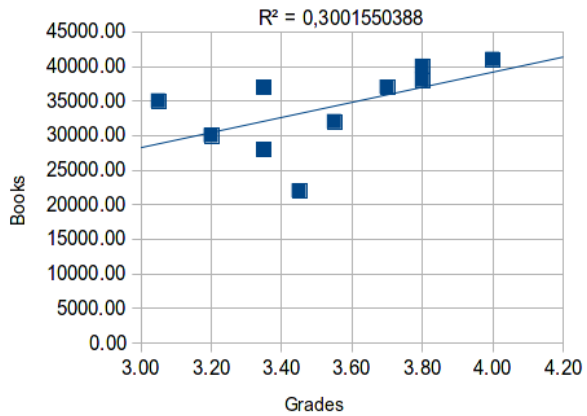
Exercises

Solution

				$x_i - \bar{x}$			$(x_i - \bar{x})^2$			$(x_i - \bar{x})(y_i - \bar{y})$		
	Grade	Scholarship	Books	Grade	Scholarship	Books	Grade	Scholarship	Books	Grade-sch	Sch-books	Grade-books
	3.05	22000.00	35000.00	-0.48	-4500.00	1000.00	0.225625	20250000	1000000	2137.5	-4500000	-475
	3.20	25000.00	30000.00	-0.33	-1500.00	-4000.00	0.105625	2250000	16000000	487.5	6000000	1300
	3.35	27000.00	28000.00	-0.18	500.00	-6000.00	0.030625	250000	36000000	-87.5	-3000000	1050
	3.35	24000.00	37000.00	-0.18	-2500.00	3000.00	0.030625	6250000	9000000	437.5	-7500000	-525
	3.45	25000.00	22000.00	-0.07	-1500.00	-12000.00	0.005625	2250000	144000000	112.5	18000000	900
	3.55	28000.00	32000.00	0.02	1500.00	-2000.00	0.000625	2250000	4000000	37.5	-3000000	-50
	3.70	28000.00	37000.00	0.18	1500.00	3000.00	0.030625	2250000	9000000	262.5	4500000	525
	4.00	30000.00	41000.00	0.48	3500.00	7000.00	0.225625	12250000	49000000	1662.5	24500000	3325
	3.80	27000.00	40000.00	0.28	500.00	6000.00	0.075625	250000	36000000	137.5	3000000	1650
	3.80	29000.00	38000.00	0.28	2500.00	4000.00	0.075625	6250000	16000000	687.5	10000000	1100
Min	3.05	22000.00	22000.00			Sum	0.80625	54500000	320000000	5875	48000000	8800
Max	4.00	30000.00	41000.00			St. dev.	0.29930475	2460.80384	5962.84794	r	0.89	0.36	0.55
Range	0.95	8000.00	19000.00							r^2	0.79	0.13	0.30
Mean	3.53	26500.00	34000.00							partial corr	0.96	-0.24	0.46
Median	3.50	27000.00	36000.00										





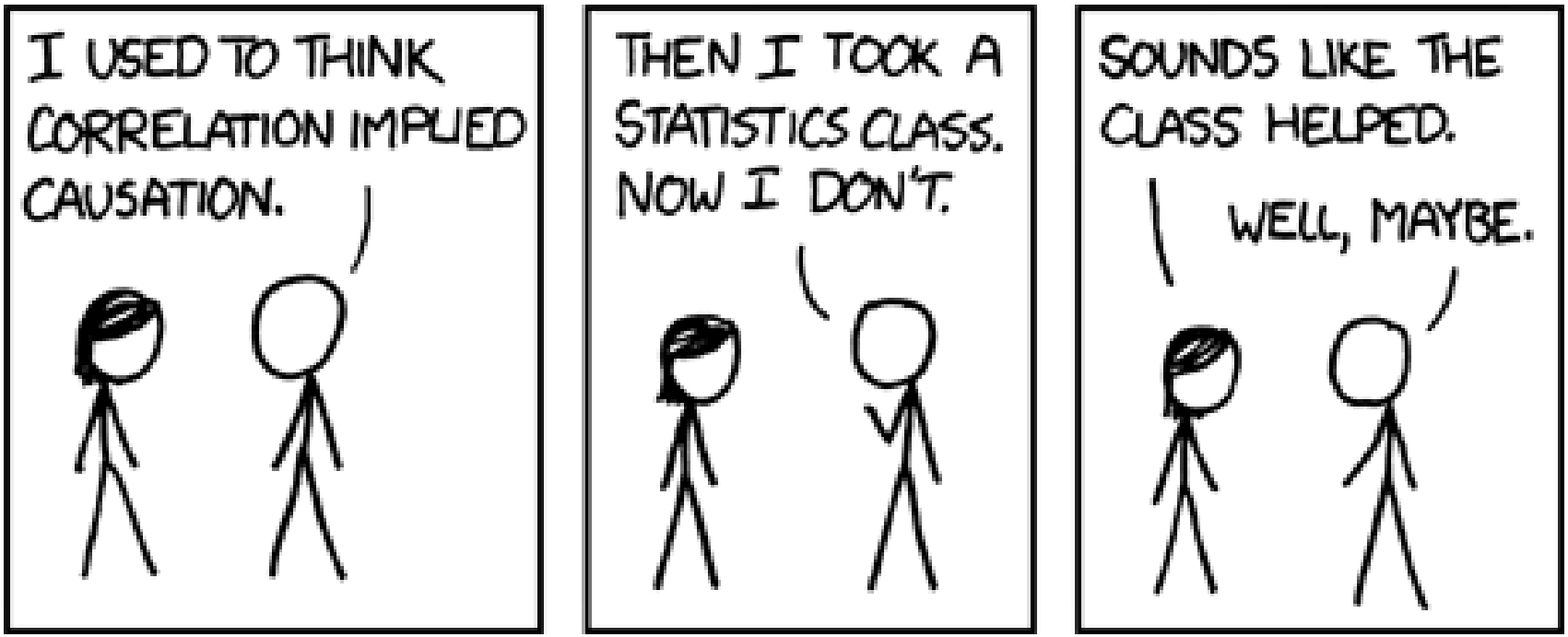


Limitations of the correlation coefficient

- Correlation and causality
- Lazarsfeld paradigm
- Correlation and linearity

Limitations of the correlation coefficient

Correlation does not imply causation!



Source: http://xkcd.com/552

Limitations of the correlation coefficient

Correlation does not imply causation! - Theoretical background

Aristotle: logic, syllogism – if $(A \rightarrow B)\&(B \rightarrow C) \Rightarrow A \rightarrow C$

David Hume: scepticism

- „only correlation can actually be perceived [not causality]”
- see: our belief that the sun will rise tomorrow
- see: „If I see a billiard ball moving towards another, on a smooth table, I can easily conceive to stop upon contact.”

Popper: falsification

Pearl, J. - *Causality: Models, Reasoning, and Inference*, Cambridge University Press, 2000

Stouffer: *The American Soldier*

Soldiers in branches with higher promotion rates are happier than soldiers in branches with lower rates of promotion.

Stouffer: *The American Soldier*

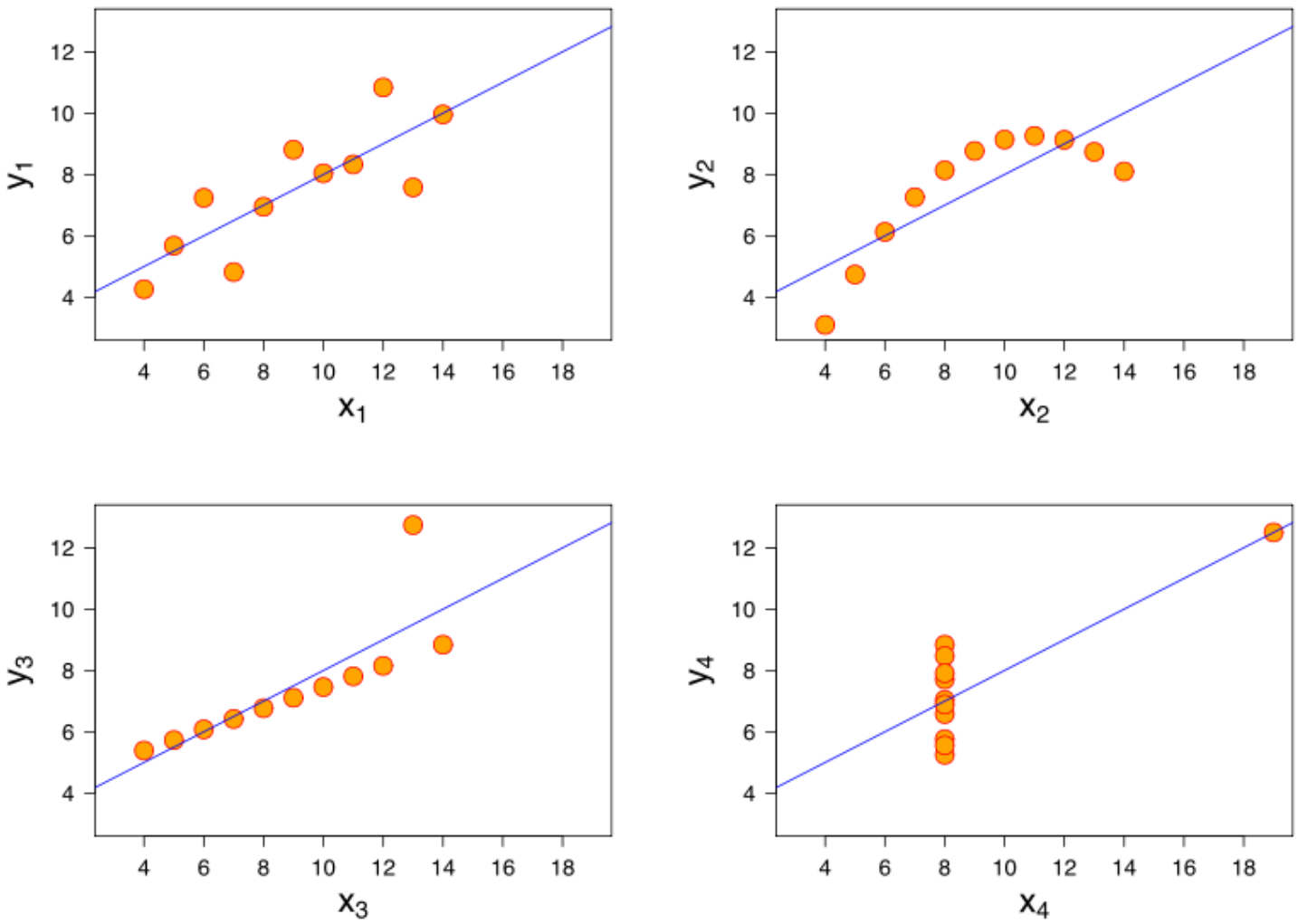
H_0: Soldiers in branches with higher promotion rates are happier than soldiers in branches with lower rates of promotion. **BUT:**

„Soldiers in branches with higher promotion rates were more pessimistic about their own chances of being promoted than soldiers in branches with lower rates of promotion."

Keywords: **reference group**, **relative deprivation**

Limitations of the correlation coefficient

Correlation and linearity - Variations of the Same Theme



Source: Anscombe, F. J. (1973) Graphs in statistical analysis. American Statistician, 27, 17–21.

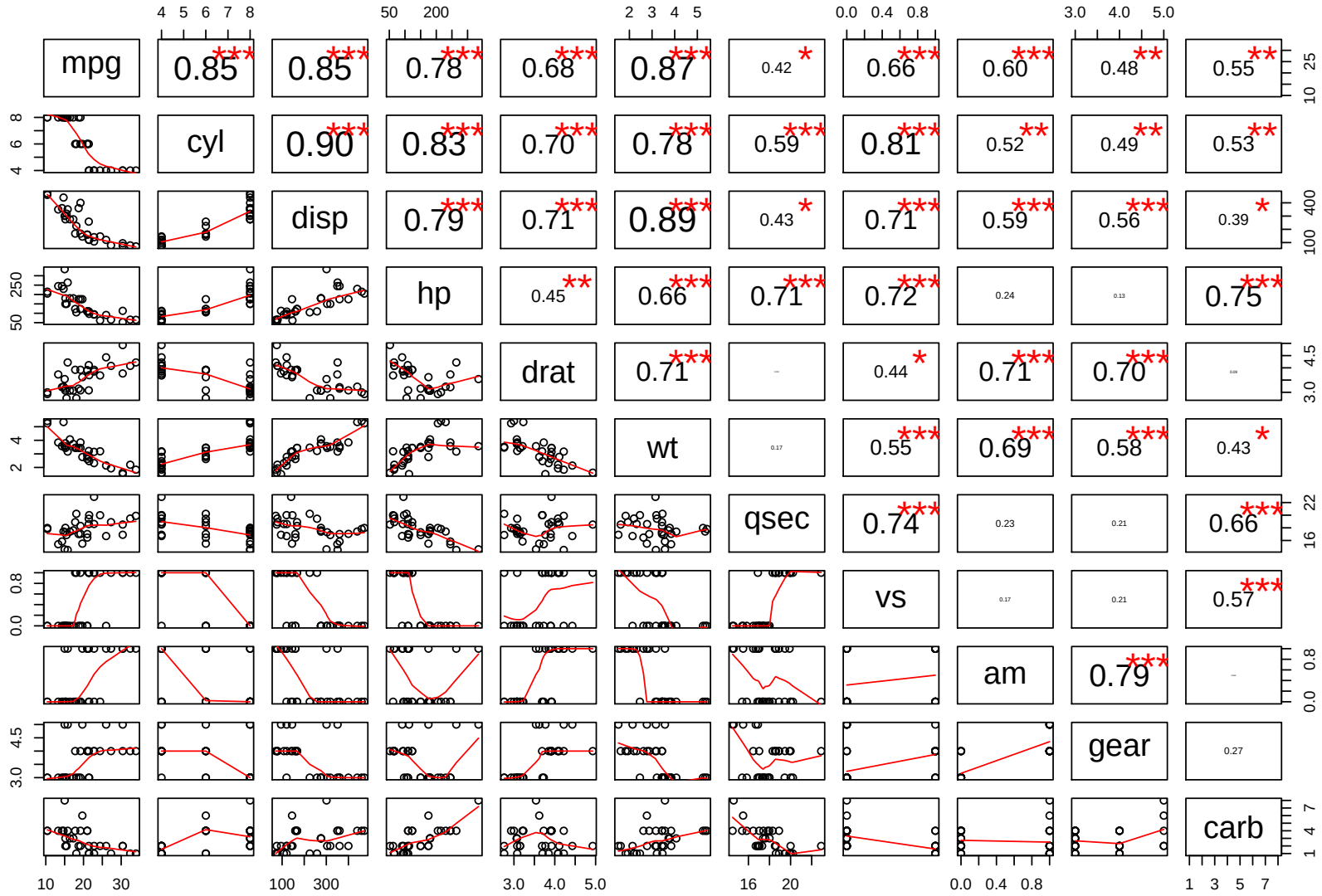
Exercise

The data was extracted from the 1974 Motor Trend US magazine, and comprises fuel consumption and 10 aspects of automobile design and performance for 32 automobiles (1973-74 models).

- mpg: Miles/(US) gallon
- cyl: Number of cylinders
- disp: Displacement (cu.in.)
- hp: Gross horsepower
- drat: Rear axle ratio
- wt: Weight (lb/1000)
- qsec: 1/4 mile time
- vs: V/S
- am: Transmission (0 = automatic, 1 = manual)
- gear: Number of forward gears
- carb: Number of carburetors

Source: Henderson and Velleman (1981), Building multiple regression models interactively. *Biometrics*, 37, 391-411.

Exercise



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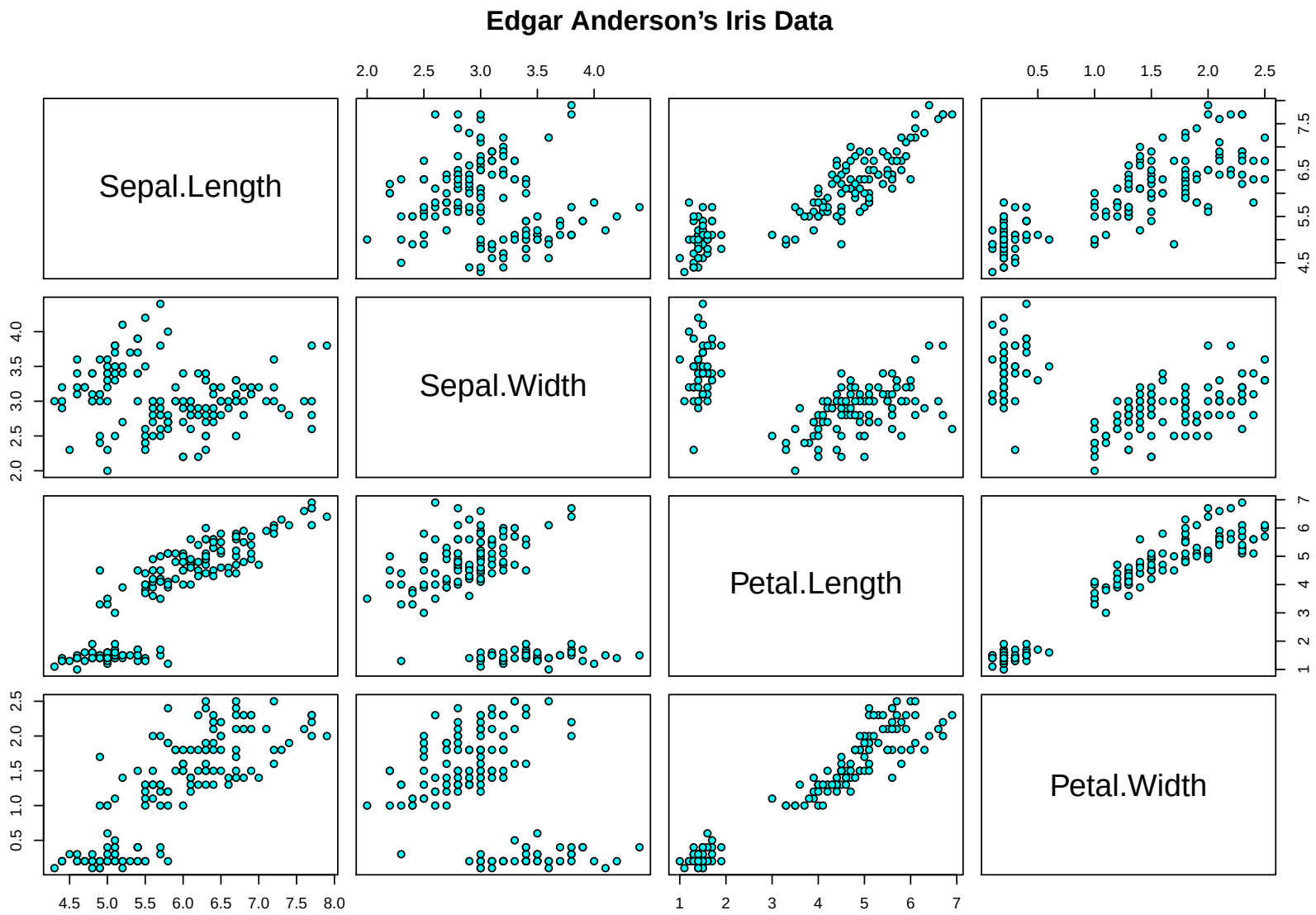

Anderson, Edgar (1935). The irises of the Gaspe Peninsula, *Bulletin of the American Iris Society*, 59, 2-5.

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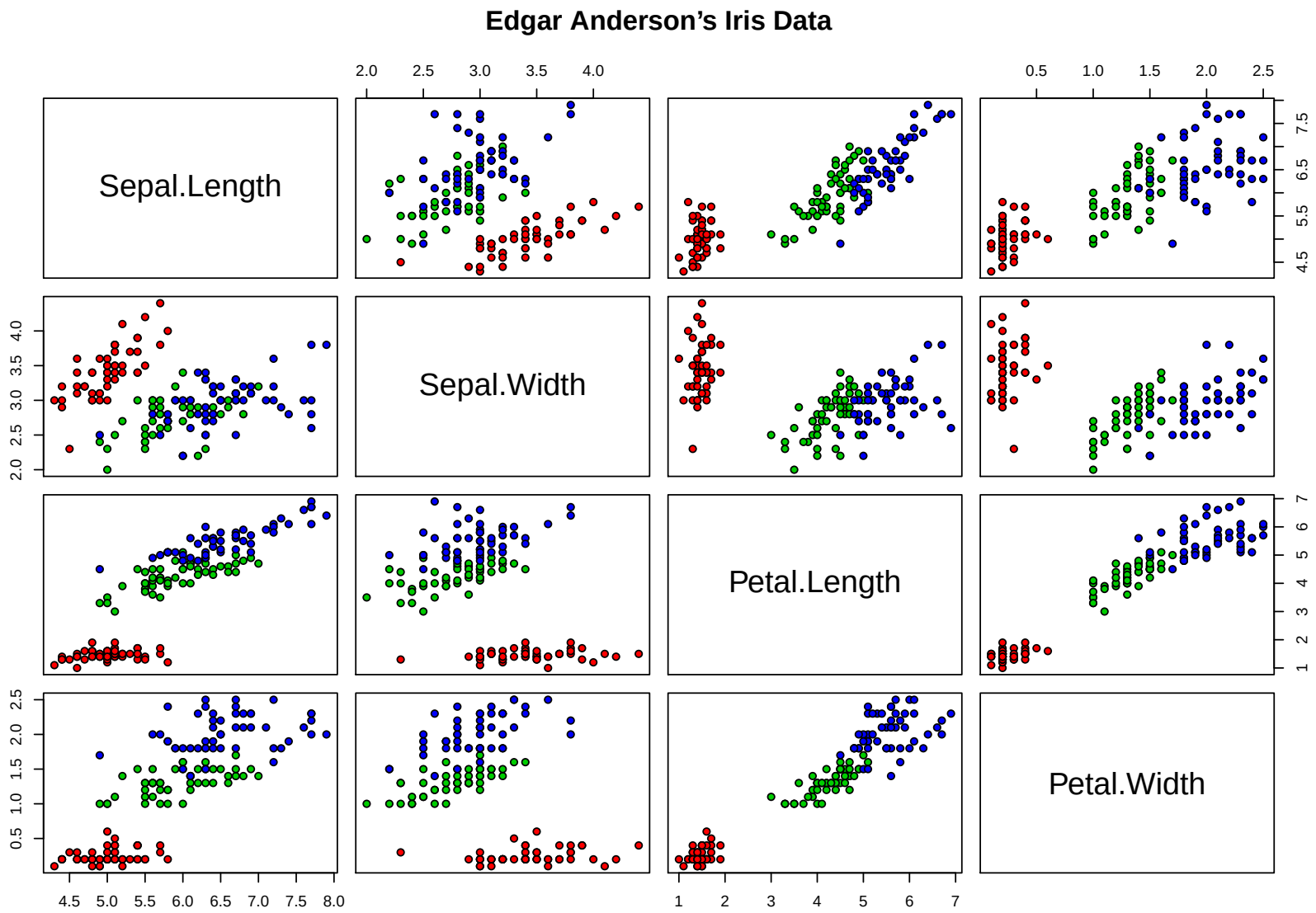

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It was a pleasure!

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